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THE C.I.E. COLORIMETRIC STANDARDS AND THEIR USE

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ABSTRACT. The new international standards, which define a standard observer, three standard illuminants, standard conditions of illuminating and viewing opaque specimens, a standard for evaluating the brightness factor of opaque specimens, and a standard trichromatic system for the expression of colour measurements, are stated and their origin explained. In addition to the numerical tables which are appended to the resolutions setting up these standards, there are given a table specifying the trichromatic coordinates for the standard observer of all spectral colours at wave-length intervals of 1 mμ, tables to facilitate the calculation of the standard coordinates and the brightness factor of a material illuminated by any one of the three standard illuminants from spectrophotometric measurements on the material, and a table giving the coordinates of some stimuli of special importance on the N.P.L. system, the standard system, and another system which occurs in the resolutions. Some new colorimetric terms are proposed, partly to avoid misinterpretation and partly to meet new needs. The theory of colour transformations, and points which arise in the application of the system and in the calibration of instruments, are discussed.

Prefatory note on nomenclature

TWO meanings of colour are firmly established in the English language. In one, the subjective sense, it denotes a quality of a visual sensation. In the other, the objective sense, it denotes an attribute of a physical stimulus which is supposed to determine its colour-matching relations to other stimuli. The word is used frequently in scientific discussions with both meanings, and at present the double use can only be avoided by substituting clumsy phrases for one of them. The ambiguity resulting from this double usage seems to have proved an obstacle to the formation of clear conceptions among scientific and technical workers, but proposals for replacing one or other of these meanings by a distinct word in technical papers have not been widely adopted. The most acceptable solution of the difficulty appears to be for writers to use colour in either sense, taking care, whenever uncertainty can arise, to specify the meaning with the precision required.

In this paper colour, both as a single word and in its compounds, is to be understood only in the objective sense, unless the contrary is stated.

Another word which has caused misunderstanding is "primary." The meaning of primary colour to an artist differs from its scientific meanings. It appears to be understood by physiologists and psychologists in a sense which it once had, but no longer has, for the physicist. In this sense primaries denote certain uniquely determinable colours by combinations of which all other colours can be matched, or alternatively they are stimuli which operate only one section of the complex mechanism through which subjective colour is presented to our consciousness. Ideas of this type no longer have a place in physics, and it seems simplest to avoid misunderstanding by the use of a different word.

When colours are expressed on a trichromatic system we are merely stating in what proportions three definite but arbitrarily chosen stimuli measured in certain units must be mixed to match this colour. We admit negative as well as positive numbers of these units. We propose, in place of primaries, to name these three stimuli the "unitary stimuli" of the particular trichromatic system, and the units of them the "trichromatic units." As we are only concerned with proportions, we need relative, not absolute, units. These units have usually been defined as the amounts of the unitary stimuli by which a fourth definite stimulus, named by Priest the "basic stimulus," has been matched. The unitary stimuli and the basic stimulus have usually been real reproducible stimuli. In this paper we have to consider a system having unitary stimuli which cannot from their nature have any physical existence, and a basic stimulus which we have no means of producing in a laboratory from its specification. The system has therefore to be defined by stimuli which are neither the unitary stimuli nor the basic stimulus.

We propose the term "cardinal stimuli" as a universally available description of the four stimuli by which any trichromatic system is defined. Thus in the N.P.L. system the cardinal stimuli are the unitary stimuli and the basic stimulus. In the new international system defined later in Resolution (5), neither the unitary stimuli nor the basic stimulus are cardinal stimuli.

PART I. THE C.I.E COLOUR STANDARDS AND THEIR DERIVATION

I. THE CONGRESS RESOLUTIONS

At the meeting of the Commission Internationale de l'Éclairage held at Cambridge in September 1931 five resolutions on colorimetry were passed. The effect of this series of resolutions is to set up for the first time an agreed basis for the expression of colour measurements, agreed sources for illuminating transparent and reflecting materials when their colours are being determined, and a standard observer for the calculation of colour coordinates from a specification of light in physical terms. The adoption of these resolutions is an event of great importance in the development of colorimetry for industrial purposes. Experimental conditions which have hitherto rested on informal understandings are now to be observed in normal general practice; the scales for recording colour measurements have every prospect of permanence; and a definite basis is provided for the coordination of different methods of colorimetry. The purpose of this paper is to show the effect of these resolutions on current colorimetric practice in Great Britain, to indicate the new paths they open, and to record the procedure followed in computing the tables and numbers incorporated in the resolutions.

The five resolutions are substantially as follows*:

Resolution (1)

It is recommended that in order to obtain a standard basis for the specification of colorimetric quantities for commercial and technical purposes such quantities be expressed with regard to a hypothetical observer, to be known as the "1931 C.I.E. Standard Observer," defined as an observer who is characterised by the C.I.E. standard visibility values and finds chromatic equivalence between the homogeneous stimuli specified by wave-lengths in Column 4 of Table I and mixtures of three homogeneous stimuli of wave-lengths 700 mμ, 546·1 mμ and 435·8 mμ in the respective proportions given in Columns 1, 2, and 3 of this table. The units are so chosen as to make a mixture of equal quantities of the three homogeneous stimuli chromatically equivalent to a heterogeneous stimulus for which the total energy of the radiation of all wave-lengths between any two limiting wave-lengths within the visible spectrum is a constant multiple of the difference between these limiting wave-lengths.

Note. The relative luminosities of the units of the three homogeneous stimuli for the standard observer are in the ratio 1 : 4·5907 : 0·0601, and the values of the ordinates of the distribution curves of these component stimuli for the so-called equal energy spectrum are given in Columns 5, 6, and 7.

Resolution (2)

It is recommended that three illuminants *A*, *B*, and *C*, as described below, be adopted as standards for the general colorimetry of materials.

A. A gas-filled lamp operated at a colour-temperature of 2848° K.

B. The same lamp used in combination with a filter consisting of a layer, one centimetre thick of each of two solutions B_1 and B_2, contained in a double cell made of colourless optical glass. The solutions are to be made up as follows:

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* The official text is in French. For convenience in use the tables with the exception of Table II are grouped together at the end of Part I of this paper; their columns have been rearranged and consequent changes introduced elsewhere.

Solution B_1:

Copper sulphate ($CuSO_4 . 5H_2O$)	2·452 g.
Mannite ($C_6H_8(OH)_6$)	2·452 g.
Pyridine (C_5H_5N)	30·0 c.c.
Water (distilled) to make	1000 c.c.

Solution B_2:

Cobalt ammonium sulphate ($CoSO_4 . (NH_4)_2SO_4 . 6H_2O$)	21·71 g.
Copper sulphate ($CuSO_4 . 5H_2O$)	16·11 g.
Sulphuric acid (sp. gr. 1·835)	10·0 c.c.
Water (distilled) to make	1000 c.c.

C. The same lamp used in combination with a filter consisting of a layer, one centimetre thick of each of two solutions C_1 and C_2, contained in a double cell made of colourless optical glass. The solutions are to be made up as follows:

Solution C_1:

Copper sulphate ($CuSO_4 . 5H_2O$)	3·412 g.
Mannite ($C_6H_8(OH)_6$)	3·412 g.
Pyridine (C_5H_5N)	30·0 c.c.
Water (distilled) to make	1000 c.c.

Solution C_2:

Cobalt ammonium sulphate ($CoSO_4 . (NH_4)_2SO_4 . 6H_2O$)	30·580 g.
Copper sulphate ($CuSO_4 . 5H_2O$)	22·520 g.
Sulphuric acid (sp. gr. 1·835)	10·0 c.c.
Water (distilled) to make	1000 c.c.

It is also recommended that the spectral energy distribution values for each of these illuminants, published in the appendix to this resolution, shall be used in the computation of colorimetric quantities from spectrophotometric measurements.

Note. It is recognised that for special purposes (for example, the specification of signal glasses) other illuminants may have to be prescribed, but, in the absence of special conditions, one of these three should be employed.

Appendix to Resolution (2)

Spectral distribution of energy of light from the sources A, B, and C

Source A. The spectral distribution of energy from this source may be taken for all colorimetric purposes to be that of a black body at a temperature of 2848° K. The value assumed for Planck's constant c_2 is 14350 micron degrees.

Source B and Source C. The spectral distributions of energy for these sources, as computed from the spectrophotometric measurements of the transmission of the filters made by Messrs Davis and Gibson of the Bureau of Standards, are tabulated below.

Table II

Wave-length in mμ	Relative energy		Wave-length in mμ	Relative energy	
	B	*C*		*B*	*C*
370	15·2	21·6	**550**	101·0	105·2
380	22·4	33·0	**560**	102·8	105·3
390	31·3	47·4	**570**	102·6	102·3
400	41·3	63·3	**580**	101·0	97·8
410	52·1	80·6	**590**	99·2	93·2
420	63·2	98·1	**600**	98·0	89·7
430	73·1	112·4	**610**	98·5	88·4
440	80·8	121·5	**620**	99·7	88·1
450	85·4	124·0	**630**	101·0	88·0
460	88·3	123·1	**640**	102·2	87·8
470	92·0	123·8	**650**	103·9	88·2
480	95·2	123·9	**660**	105·0	87·9
490	96·5	120·7	**670**	104·9	86·3
500	94·2	112·1	**680**	103·9	84·0
510	90·7	102·3	**690**	101·6	80·2
520	89·5	96·9	**700**	99·1	76·3
530	92·2	98·0	**710**	96·2	72·4
540	96·9	102·1	**720**	92·9	68·3

Resolution (3)

For the purpose of colorimetric measurements of reflecting materials, except where special circumstances dictate otherwise, the illumination should be incident at an angle of 45° and the line of observation should be normal to the surface of the specimen.

Where the special nature of the material or the special objects of an investigation require a departure from these conditions, a statement of the actual conditions employed should always be included with any published results.

Resolution (3 *a*)

In colorimetric measurements of opaque materials, the brightness of the material shall be expressed relative to the brightness of a surface of magnesium oxide under the same conditions of illumination.

Resolution (4)

The standard system of colorimetric specification shall be a system in which the colour-quality of any stimulus is expressed on three scales which are determined by giving prescribed values to four selected stimuli so chosen that no two can be combined to match a third.

Resolution (5)

The four stimuli which define the standard colorimetric scales shall consist of homogeneous radiations of wave-lengths 700 mμ, 546·1 mμ, and 435·8 mμ, and the radiation from standard illuminant *B*. These stimuli shall be assigned the following values:

$0{\cdot}73467\mathbf{X} + 0{\cdot}26533\mathbf{Y} + 0{\cdot}00000\mathbf{Z}$ for 700 mμ
$0{\cdot}27376\mathbf{X} + 0{\cdot}71741\mathbf{Y} + 0{\cdot}00883\mathbf{Z}$ for 546·1 mμ
$0{\cdot}16658\mathbf{X} + 0{\cdot}00886\mathbf{Y} + 0{\cdot}82456\mathbf{Z}$ for 435·8 mμ
$0{\cdot}34842\mathbf{X} + 0{\cdot}35161\mathbf{Y} + 0{\cdot}29997\mathbf{Z}$ for standard illuminant *B*.

Note. The properties of the standard observer, when expressed on the system thus defined, are given in Table III.

II. DERIVATION AND DISCUSSION OF THE STANDARDS

We consider each of these resolutions in turn.

Resolution (1)

It is remarkable that, despite the clear need for a precise determination of the relations between the colours of light at different parts of the visible spectrum, many years have elapsed between the realisation of this need and the completion of any investigation planned to meet it. In the absence of more satisfactory information, limited use has been made of the results of König and Dieterici, both in the modified form due to Ives (1), and in that due to Weaver (2), who took into account Abney's results for a portion of the spectrum. It has not, however, been possible to adopt these values as a general basis for coordinating diverse methods of colour measurement. For some time it has been recognised with increasing clearness that new measurements of greater precision were required to facilitate new technical developments.

Within the last five years two experimental determinations of these relations have been made. In 1926–7 an investigation was carried out by Guild at the National Physical Laboratory with the aid of seven observers whose colour vision was found to be free from any marked abnormality. The purpose of this investigation was to facilitate the standardisation of industrial colour measurements, and special attention was given to the conditions which are likely to be encountered in this work. Particular care was taken to base the calibration of the apparatus on direct observations of reproducible light sources.

In the following year a similar investigation was carried out for the British Medical Research Council by Wright (3) at the Imperial College of Science. He employed ten observers for a complete investigation and obtained supplementary observations from a further thirty. The aims of this investigation were somewhat wider than Guild's, but the methods of experiment were similar in principle, and the attention paid to the standardisation of the auxiliary equipment, and the adoption of standard conditions of observation, render the results of immediate value in standardised colorimetry.

The paper (4) describing Guild's investigation, which appeared later than Wright's, contained a proposal that the mean results of these two investigations should be taken to define a standard observer for industrial colorimetry.

After this proposal had appeared, it was learnt that an extensive statistical study of colour vision had been planned and begun in 1926 by Kohlrausch and others. The programme included a re-determination of spectral values on the lines of the earlier investigation of König and Dieterici. A preliminary account of the method employed, with his own measurements, is given in a recent article by Kohlrausch (5). Determinations by other observers appear not to have begun when the article was written. Kohlrausch's experimental arrangements, and the method of deducing relations from the observed colour matches, resemble those used by König and Dieterici. The procedure is less direct than that of Guild and Wright. The work also differs in aim, for the interest is mainly physiological. While the influence of adaptation and other visual conditions is discussed in detail, the particulars given of experimental technique, standardisation of light sources, calibration of the nicol system and other parts of the apparatus, are insufficient for the reliability of the work from the metrological point of view to be evaluated.

The only completed investigations which have yielded results directly applicable to industrial colorimetric standardisation are thus those of Guild and Wright. So far as is known no other comparable investigations are at present contemplated. On these grounds the British National Committee for Colorimetry adopted Guild's proposal for the specification of a standard observer. The comments received from other National Committees on this proposal, and the discussions which followed, led to the ultimate presentation to the Commission of Resolution (1), which embodies these results but expresses them in a different form.

Originally Guild proposed the adoption of the arithmetic mean of his and Wright's results, on the colorimetric scales of what is now known as the N.P.L. system. In this system a colour is expressed by the proportions in which radiation of three particular wave-lengths must be combined to match the colour under measurement. This, like any other system based on the mixture of three spectral colours, involves the presence of negative coefficients in the specification of any other spectral colour and of some non-spectral colours. American opinion is unfavourable to the use of negative numbers, and it became clear that to secure agreement all the quantities would have to be transformed to a new system. It was decided to take advantage of the change to meet minor criticisms made both in this country and abroad. Alterations were called for owing to a numerical error in one of Wright's figures, and to other irregularities present in the unsmoothed mean of Guild's and Wright's results. All these irregularities were removed in the initial smoothing process, which was applied to the figures defining Guild's and Wright's spectral loci on the N.P.L. system.

The operation of smoothing under the abnormal conditions present in this problem is less simple than might be thought. Fig. 1 shows how the values finally reached differ from Guild's values on the one hand and from Wright's on the other. The final values are smooth, and lie as evenly between the two determinations as can be expected.

As the spectral locus reached by this process differs slightly from those previously considered, it became necessary to see whether the luminosity factors ascribed to the trichromatic units of the three unitary stimuli would have to be modified. Calculation with the standard visibility function showed that the assumed values were in good agreement with the experimental determinations.

For reasons which led to the substitution of new standard illuminants in place of the "N.P.L. standard white light," the specification of the standard observer on a system defined in part by this source became inappropriate. As will be explained later, it was decided to recommend the use of a metric system so contrived that the standard observer would represent the colour of an equi-energy source—a purely theoretical energy distribution—by the centroid of the colour triangle. The new system was not otherwise yet agreed. As a practical step the units of the N.P.L. system were altered to bring the colour corresponding to this particular energy distribution to the centroid of the triangle, while retaining at its vertices the three spectral colours of the N.P.L. system.

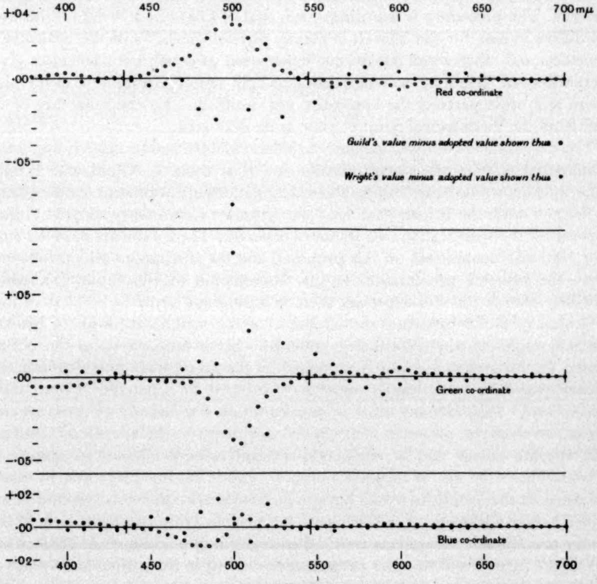

Fig. 1.

The nature of the calculation will be more readily understood if we introduce a few symbols. On the N.P.L. system the unitary stimuli are homogeneous radiations of the three wave-lengths 700 mμ, 546·1 mμ, and 435·8 mμ respectively. The units of these stimuli, so chosen that a mixture of one unit of each of the three matches the N.P.L. white, are denoted by R', G', and B' respectively. The relative luminosities of the units are in the ratio $1 : L_G' : L_B'$. On the system of the first resolution the units of the same stimuli are denoted by R, G, and B, and the ratio of their luminosities by $1 : L_G : L_B$. The standard visibility function is denoted by V, and a subscript is added to distinguish its values for a particular wave-length when desired. Thus V_λ is the value of V for wave-length λ. By the definition of V, $E_\lambda V_\lambda d\lambda$ is the luminosity of radiation of total energy $E_\lambda d\lambda$, the wave-lengths lying between the limits $\lambda \pm \frac{1}{2}d\lambda$, where $d\lambda$ is small in comparison with λ.

Now from the smoothed spectral figures derived from Guild's and Wright's observations, we know that on the N.P.L. system one trichromatic unit of light of wave-length λ matches a certain mixture of the units R', G', B' in both colour and brightness. If this trichromatic unit is denoted by C_λ', the match is expressed by the equations

$$C_\lambda' = r_\lambda' R' + g_\lambda' G' + b_\lambda' B', \quad r_\lambda' + g_\lambda' + b_\lambda' = 1$$

where r_λ', g_λ', b_λ' are tabulated numbers or numbers derived from them by interpolation. The luminosity of C_λ' is thus

$$r_\lambda' + g_\lambda' L_G' + b_\lambda' L_B',$$

and of n similar trichromatic units is

$$n\,(r_\lambda' + g_\lambda' L_G' + b_\lambda' L_B').$$

If the interval $d\lambda$ is sufficiently small this expression will also represent the luminosity of n units of almost homogeneous light comprising radiation of wave-lengths lying between the limits $\lambda \pm \frac{1}{2}d\lambda$. Let us suppose that there are n trichromatic units in the radiation $E_\lambda d\lambda$. Then

$$n\,(r_\lambda' + g_\lambda' L_G' + b_\lambda' L_B') = E_\lambda V_\lambda d\lambda$$

or

$$n = \frac{V_\lambda}{r_\lambda' + g_\lambda' L_G' + b_\lambda' L_B'} E_\lambda d\lambda = \frac{V_\lambda}{L_\lambda'} E_\lambda d\lambda.$$

Thus the radiation $E_\lambda d\lambda$ gives rise to exactly the same sensations as a mixture of

$$\frac{r_\lambda' V_\lambda}{L_\lambda'} E_\lambda d\lambda, \quad \frac{g_\lambda' V_\lambda}{L_\lambda'} E_\lambda d\lambda, \quad \frac{b_\lambda' V_\lambda}{L_\lambda'} E_\lambda d\lambda$$

of the trichromatic units R', G', B' respectively. Now let us suppose we have a source emitting radiation throughout the whole visible spectrum, the energy in the neighbourhood of wave-length λ being given by the function E_λ. Then the whole of the visible radiation emitted by this source will match a combination of

$$\int \frac{r_\lambda' V_\lambda}{L_\lambda'} E_\lambda d\lambda, \quad \int \frac{g_\lambda' V_\lambda}{L_\lambda'} E_\lambda d\lambda, \quad \int \frac{b_\lambda' V_\lambda}{L_\lambda'} E_\lambda d\lambda$$

of the units R', G', B' respectively, where the integrals are taken over the whole range of wave-lengths for which V is not zero. Now let ϵ represent radiation from a source for which E_λ is a constant independent of λ (i.e. a source emitting the so-called equi-energy spectrum). Then ϵ yields a colour match with the mixture

$$\int \frac{r_\lambda' V_\lambda}{L_\lambda'}\, d\lambda . R' + \int \frac{g_\lambda' V_\lambda}{L_\lambda'}\, d\lambda . G' + \int \frac{b_\lambda' V_\lambda}{L_\lambda'}\, d\lambda . B'.$$

We now derive the system used in specifying the properties of the standard observer in this resolution by taking new units R, G, B of radiation of the same wave-lengths as R', G', B' according to the definitions

$$\frac{R}{R'} = c \int \frac{r_\lambda' V_\lambda}{L_\lambda'}\, d\lambda, \quad \frac{G}{G'} = c \int \frac{g_\lambda' V_\lambda}{L_\lambda'}\, d\lambda, \quad \frac{B}{B'} = c \int \frac{b_\lambda' V_\lambda}{L_\lambda'}\, d\lambda,$$

where c is an arbitrary constant. The new coordinates or colour valencies $r_\lambda, g_\lambda, b_\lambda$ of spectral radiation of wave-length λ are therefore connected with the old coordinates $r_\lambda', g_\lambda', b_\lambda'$ as shown in the equations

$$r_\lambda = c' \frac{r_\lambda'}{\int \frac{r_\lambda' V_\lambda}{L_\lambda'}\, d\lambda}, \quad g_\lambda = c' \frac{g_\lambda'}{\int \frac{g_\lambda' V_\lambda}{L_\lambda'}\, d\lambda}, \quad b_\lambda = c' \frac{b_\lambda'}{\int \frac{b_\lambda' V_\lambda}{L_\lambda'}\, d\lambda},$$

where c' is to be found for each wave-length from the condition

$$r_\lambda + g_\lambda + b_\lambda = 1.$$

The ratios of the new relative luminosity factors L_G and L_B to the old factors are determined by the changes in the units of the unitary stimuli. Thus

$$\frac{L_G}{L_G'} = \frac{\int \frac{g_\lambda' V_\lambda}{L_\lambda'}\, d\lambda}{\int \frac{r_\lambda' V_\lambda}{L_\lambda'}\, d\lambda}, \quad \frac{L_B}{L_B'} = \frac{\int \frac{b_\lambda' V_\lambda}{L_\lambda'}\, d\lambda}{\int \frac{r_\lambda' V_\lambda}{L_\lambda'}\, d\lambda}.$$

The equi-energy stimulus ϵ is represented on the new system by the centroid of the colour triangle, for it is clear that the effect of the changes in the units we have just made is to cause it to satisfy the colour equation

$$\epsilon = \tfrac{1}{3}R + \tfrac{1}{3}G + \tfrac{1}{3}B.$$

The calculations described, preparatory to the change of units, are typical of the procedure adopted in computing colour coordinates from a knowledge of the function E defining the distribution of the energy emitted by a source for the whole of the visible spectrum. Just as on the old system we evaluated the quantities

$$\frac{r_\lambda' V_\lambda}{L_\lambda'}, \quad \frac{g_\lambda' V_\lambda}{L_\lambda'}, \quad \frac{b_\lambda' V_\lambda}{L_\lambda'},$$

so in the corresponding calculations on the system of this resolution we require the quantities

$$\bar{r}_\lambda = \frac{r_\lambda V_\lambda}{L_\lambda}, \quad \bar{g}_\lambda = \frac{g_\lambda V_\lambda}{L_\lambda}, \quad \bar{b}_\lambda = \frac{b_\lambda V_\lambda}{L_\lambda},$$

where $$L_\lambda = r_\lambda + g_\lambda L_G + b_\lambda L_B.$$

These quantities are proportional to the ordinates of the curves which have been named the spectral mixture curves (or the spectral distribution curves) of the unitary stimuli for the equi-energy spectrum. They are given in Columns 5, 6, and 7 of Table I for values of λ at intervals of 5 mμ.

In practice the integrals are replaced by the sums of the values of the integrand at suitable intervals of λ. In the calculations just described, no significant difference in the results would follow from employing a smaller interval than $d\lambda = 5$ mμ.

Resolution (2)

In the N.P.L. scheme only one standard illuminant was used. It was recognised that this illuminant could not be adopted for all measurements: the use of another special illuminant is an essential part of some of the standardisation already carried out. The very fact that some materials appear of quite different colours when seen in daylight and by the light of electric filament lamps shows conclusively that measurements must at times be made with various illuminants. There appeared at one time to be some prospect that one particular source could be used much more widely than any other without serious inconvenience, provided it represented a fair mean between the illuminants of major importance. By adopting only one standard illuminant it was hoped to establish the practice of taking this illuminant for granted, unless the use of a different source as illuminant was specifically mentioned. This practice does not appear to have been adopted in any other country than Great Britain.

In the exchange of views which took place before the Cambridge meeting, some emphasis was laid on the desirability of securing uniformity in practice when sources of both higher and lower colour temperatures than that of the N.P.L. source were used. For this reason three illuminants have now been standardised, and as no agreement has been reached that one of the three is to be recommended for use whenever possible in preference to either of the other two, it becomes necessary in publishing the results of colour measurements to specify in every case which illuminant has been employed. As the advantages of international standardisation would be largely lost if it became the common usage to employ only one of these illuminants in one country and a different one in another country, it is considered desirable that, in every industry in which colour measurements of transparent or opaque materials are important, experience should be gained with each of the three standard illuminants. It is possible that extensive trials may show that one of the three is of much greater practical value and convenience for general purposes than either of the other two. If this conclusion were reached, industrial colour measurements made in different countries would tend to become strictly comparable, and a specification prepared in one country would be unambiguous to manufacturers of another country.

As it is only possible with unscreened incandescent lamps to reach the lower end of the range of colour temperatures of general importance in colour measurements, filters must be used in obtaining the two bluer sources. If, as is convenient, the same materials are to be used in the construction of both these filters, there is at present no alternative to the use of Davis-Gibson filters. Two filters of a series designed for use with a gas-filled lamp operated at colour temperatures of 2848° K. were accordingly chosen, one to yield a source of colour temperature about 6500° K., which represents one of the bluer phases of daylight, the other to yield a source of colour temperature about 4800° K., corresponding to one of the yellower phases of daylight. For convenience the unscreened gas-filled lamp itself serves as the third source. The particular colour temperature 2848° K., rather than the round figure 2850° K., appears in the specification owing to coincidences in measurements at various American laboratories on the colour temperature of the source used at the Bureau of Standards in the development of these filters.

The source of colour temperature 4800° K. is very similar to the N.P.L. standard white light in its energy distribution, but not identical with it. When it is important that results obtained with the new illuminants should be as closely comparable as possible with those derived in the past on the N.P.L. system (e.g. in tests of fugitiveness on established fading scales*), standard illuminant *B* should be employed. When special considerations of this kind are lacking, the investigator should decide which of the three illuminants is the most suitable for the work in hand. In case of doubt he will be well advised, as has been said earlier, to obtain experience with all three as far as circumstances permit.

The spectral energy distributions of the sources *B* and *C* which are tabulated in the Appendix to Resolution (2) are those published by Davis and Gibson, and these values are to be used in calculating colour-coordinates from spectrophotometric measurements. As the published values are at wave-length intervals of 10 mμ, it is necessary to supplement them by interpolated values at intermediate wave-lengths. No interpolates appear in the official recommendations, as none were available at the time of the Congress. The omission is of little importance as the values reached by any of the regular methods of interpolation will not differ by amounts likely to be of colorimetric significance.

The interpolates which we have prepared for use at the National Physical Laboratory are given in Table V. This set of values will probably be found suitable for general use until revised figures for the three illuminants are adopted at a future Congress.

Davis and Gibson's paper contained values for illuminant *A* involving four significant figures, and for illuminants *B* and *C* three significant figures. In their colorimetric application these values are multiplied by factors usually having four significant figures. Both factors should be specified to the same order of accuracy. A final zero has therefore been added to each of Davis and Gibson's figures for illuminants *B* and *C* and the interpolates computed to four figures. It would have been possible to attain greater smoothness in the function these figures define by adding quantities within the limits ± 0·05 to Davis and Gibson's figures, but the course adopted was thought to be preferable. The interpolated values were computed by the Lagrangian method using the six nearest tabulated quantities. The differences between four- and six-point interpolation were invariably very small. The tables were extended for wave-lengths exceeding 720 mμ by adopting values sent by the Bureau of Standards, Washington.

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* For materials that may be classed as only moderately selective the colour coordinates found with the N.P.L. white light and those obtained with standard illuminant *B* should be substantially equal provided the scales of the colorimeter are also calibrated by observations on this source, as discussed in Section VIII. Most of the materials for which colour standards have been set up in this country probably belong to this class.

On tabulating Davis and Gibson's values for the black body at 2848° K. it was noticed that irregularities, clearly indicative of appreciable errors, were present. The values given for source A in Table V were therefore computed directly from Planck's law, using the value of c_2 adopted in the derivation of Davis and Gibson's figures. Only five of the numbers, in addition to the conventional value 100·00 at 560 mμ, of the two determinations agree. The average difference is nearly 0·05 and the greatest differences exceed four times the mean difference.

It is perhaps worth pointing out that Lagrangian interpolation is both convenient and advantageous when computations such as those required in colorimetry are concerned. If we define the function f by means of a numerical table for integral values of its variable x between certain limits, and if f and all its derivatives vanish outside these limits, the integral $\int f\,dx$ taken over the range for which f is not zero is given accurately by summing the tabulated values of f, provided the function is defined for non-integral values of x by Lagrangian interpolation of any given order. By postulating Lagrangian interpolation we therefore remove all uncertainty from the practical determination of the integrated quantities in which we are interested, provided the accuracy of the tabulated values is not open to doubt.

Resolutions (3) *and* (3*a*)

These standard conditions for viewing and illuminating opaque specimens represent the practice adopted in the N.P.L. system. They were reached after experiments had been carried out on materials of the most varied character with surfaces exhibiting markedly different properties. So far as is known these conditions have been found entirely satisfactory in the industrial and research colorimetric laboratories in this country.

The use of the magnesium oxide diffusing surface for the evaluation of the brightness factors of opaque materials is also in accordance with current British practice.

Recent determinations by Preston (6) indicate that under the specified conditions of illumination and observation the effective reflection factor of magnesium oxide, deposited on silver or other white metal of high reflectivity, is accurately reproducible and only differs by trivial amounts from unity. The difference between a brightness factor relative to magnesium oxide and the corresponding absolute brightness factor is therefore negligible for industrial colorimetry under the conditions prescribed in Resolution (3).

Resolution (4)

This resolution records the adoption, for international colorimetric standards, of two principles which are already familiar to British colorimetrists. In the first place it is agreed that of the various ways of specifying a colour the trichromatic method must be taken as the standard method. Alternative methods of specification, for example the monochromatic-plus-white method, or methods used in proprietary systems, however useful they may be, do not provide an adequate basis for general scientific colorimetry. The trichromatic method alone provides a basis on which colours can be adequately related to the physical properties of light sources and materials, and all other systems correlated with one another. The metrological bases of all other systems can be regarded as mere derivatives of the trichromatic method of classification. Secondly it is recognised that the correct way of specifying a trichromatic system is to assign definite scale values to four real reproducible standard stimuli, the cardinal stimuli.

Any reproducible colours and any coordinates, provided any selection of three of them are independent*, are theoretically admissible. All the properties of the unitary stimuli and the relative magnitude of the units are uniquely determinable from them. The unitary stimuli may be real (as in the N.P.L. system, where they and the basic stimulus are the cardinal stimuli of the system), or unreal, in the sense that they can have no independent existence, but only occur in limited combinations which match real stimuli. On the colour triangle real stimuli are confined to the convex region bounded by the spectral locus and the straight line joining its extremities.

Resolution (5)

It has already been mentioned that a desire had been expressed for the standardisation of a system in which all the coordinates of every real colour are positive. This condition is not satisfied by the N.P.L. system, and another had therefore to be sought. An indefinite number of systems may be constructed for the expression of colorimetric relations, but, though in theory all may be regarded as equivalent to one another, they are by no means all on an equality in convenience for dealing with practical problems. In comparing systems the principal points to be considered are

(1) the nature of the unitary stimuli,
(2) the accuracy with which a given number of figures specify colours,
(3) the amount of work involved in indirect colorimetry.

We note that the requirement for all-positive coordinates for every real stimulus involves unreal properties for at least one of the unitary stimuli. (By an unreal stimulus we mean that no stimulus with these colorimetric properties can be produced.) We also observe that no system of this type which is satisfactory on the second point can be obtained unless at least two of the unitary stimuli are unreal†. We therefore only have to consider systems with either one or no real unitary stimuli. If a real stimulus is used it is necessarily the light from one of the extremities of the visible spectrum, and two of the coordinates for the final wave-length are then zero. If the red end of the spectrum is chosen it happens that these zero values are maintained for an appreciable range of wave-lengths, and hence zeroes also occur in the associated quantities used in computing colours from the physical specification of a stimulus, since the colour coordinates are factors of these quantities. The labour of these calculations is therefore diminished. Additional zeroes can be secured for one coordinate by making a side of the colour triangle tangential to the spectral locus at this point.

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* Three colours are independent when no two form a match with the third: their coordinates are independent if they do not satisfy the condition for collinearity.

† The considerations on which these statements are based are not given here, as they are not essential to the main purpose of this paper. They are discussed in the Appendix.

There is another way in which colorimetric calculations can be simplified. When a stimulus is defined in physical terms it is usually desired to determine the brightness factor in addition to the colour coordinates. This involves a small subsidiary calculation, but by adopting a particular device suggested by Judd (7) this calculation becomes unnecessary. Since we are adopting unreal units we may consider units having properties entirely different from those of any real stimulus. We may imagine units which have colour but no visibility. Judd's suggestion is to adopt two units of zero visibility, so that the visibility depends only on the third unit, the other two merely modifying the colour. Now on the system of Resolution (1) the units R, G, B have relative visibilities in the ratio $1 : L_G : L_B$, where L_G and L_B are definite numbers. The visibility of one trichromatic unit of the colour having the coordinates r, g, b is therefore $r + gL_G + bL_B$, and if we equate this expression to zero we obtain the locus of colours of zero visibility. It is called the alychne, and is shown in Fig. 2 by the straight line XZ. If we choose two unitary stimuli which are represented by points on this line we satisfy Judd's condition.

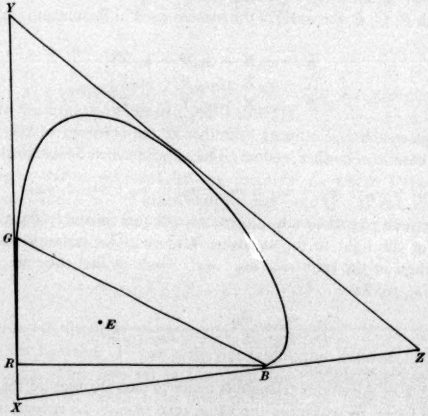

Fig. 2.

We cannot obtain a system which meets the second requirement satisfactorily by choosing a triangle with two points on the alychne and the third at an extremity of the visual spectral locus. The utmost we can do is to satisfy one of these conditions. The choice made consists in taking the alychne for one side XZ of the colour triangle and the tangent to the spectral locus at the red end of the spectrum for another side XY, and arranging the third side YZ to pass near to the spectral locus, without touching or cutting it, in a direction which secures satisfactory metrical properties in the new system. The precise procedure followed in deriving this system will now be described.

We denote the three units of the system to be found by the black letters $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$. All that is known about them at present is that they are unreal. These units will be connected with R, G, B, the units of the system used in Resolution (1), by equations of the form

$$\left.\begin{aligned} R &= \omega_{11}\mathbf{X} + \omega_{12}\mathbf{Y} + \omega_{13}\mathbf{Z} \\ G &= \omega_{21}\mathbf{X} + \omega_{22}\mathbf{Y} + \omega_{23}\mathbf{Z} \\ B &= \omega_{31}\mathbf{X} + \omega_{32}\mathbf{Y} + \omega_{33}\mathbf{Z} \end{aligned}\right\} \qquad \ldots\ldots(1).$$

We wish to place the equi-energy stimulus at the centroid of the new triangle. It is already there in the other system. The conditions to be satisfied are therefore

$$\omega_{11} + \omega_{21} + \omega_{31} = \omega_{12} + \omega_{22} + \omega_{32} = \omega_{13} + \omega_{23} + \omega_{33},$$

and it is convenient to make each of these sums equal to unity. Now let us ascribe the visibility of the light to the stimulus $\mathbf{Y}$ alone. The luminosities of the units R, G, B are then in the ratio $\omega_{12} : \omega_{22} : \omega_{32}$. Since in fact they are in the known ratio $1 : L_G : L_B$, we have

$$\frac{\omega_{12}}{1} = \frac{\omega_{22}}{L_G} = \frac{\omega_{32}}{L_B} = \frac{1}{1 + L_G + L_B},$$

so that ω_{12}, ω_{22}, ω_{32} are found.

The side XY of the colour triangle is made to pass through R, the red terminus of the spectral locus, by assigning the value zero to ω_{13}. To extend the number of zero coordinates as far as possible, we make XY tangential to the spectral locus at R. This condition determines the ratio of ω_{23} to ω_{33}, and as their sum is unity the three coefficients of $\mathbf{Z}$ are found. The third side is determined by the values of ω_{11}, ω_{21}, ω_{31}. The conditions are that it should pass the spectral locus at a reasonable distance, and lie in a direction which gives a favourable disposition of the spectral locus within the triangle. The coefficients determined in this way are

$$\left.\begin{aligned} \omega_{11} &= 0{\cdot}49000, & \omega_{12} &= 0{\cdot}17697, & \omega_{13} &= 0{\cdot}00000 \\ \omega_{21} &= 0{\cdot}31000, & \omega_{22} &= 0{\cdot}81240, & \omega_{23} &= 0{\cdot}01000 \\ \omega_{31} &= 0{\cdot}20000, & \omega_{32} &= 0{\cdot}01063, & \omega_{33} &= 0{\cdot}99000 \end{aligned}\right\} \qquad \ldots\ldots(2).$$

On the system of Resolution (1) the colour associated with light of a given wavelength is determined by the tabulated coefficients r, g, b, and the corresponding colour equation is

$$Q = rR + gG + bB.$$

To obtain the equation for this colour on the new system we first have to substitute for R, G, B in terms of $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ from equations (1), using the numerical values (2). In this way we obtain

$$Q = x'\mathbf{X} + y'\mathbf{Y} + z'\mathbf{Z},$$

where

$$\begin{aligned} x' &= r\omega_{11} + g\omega_{21} + b\omega_{31}, \\ y' &= r\omega_{12} + g\omega_{22} + b\omega_{32}, \\ z' &= r\omega_{13} + g\omega_{23} + b\omega_{33}. \end{aligned}$$

In general the sum of x', y', and z' is not unity, so that Q does not represent a trichromatic unit on the new system. The unit equation is

$$\mathbf{Q} = x\mathbf{X} + y\mathbf{Y} + z\mathbf{Z},$$

where

$$\frac{x}{x'} = \frac{y}{y'} = \frac{z}{z'} = \frac{1}{x' + y' + z'}.$$

The values of x, y, z derived in this way from the values of r, g, b given in Table I are recorded in the first three columns of Table III.

In the computation of colour coordinates from a knowledge of the spectral energy distribution by the method described in the discussion of Resolution (1) we require to weight the coordinates by the factor V/L, where V is the standard visibility function and L is the luminosity of the unit $\mathbf{Q}$. As the visibility of the radiation is confined to the unit $\mathbf{Y}$, the luminosity of $\mathbf{Q}$ is measured by its coefficient y. The quantities required are therefore

$$\bar{x} = \frac{x}{y}V, \quad \bar{y} = V, \quad \bar{z} = \frac{z}{y}V.$$

They are recorded in the last three columns of Table III.

The visibility function V has only been officially defined for wave-length intervals of 10 mμ. This interval is too great for colorimetric calculations. As satisfactory interpolated values have been published by Judd (8) these have been adopted. Values have also been added outside the range for which V has been tabulated to define adequately the way in which the function falls to zero at the extremities of the visible spectrum.

All the original calculations were carried out to seven decimal places, and tests of a searching character were applied to ensure the accuracy of the figures finally reached. The reduction to four significant figures was only made after the reliability of the seven figure results was considered to be fully established. A feature peculiar to unit equations arises when the number of figures is reduced. If the four figure numbers nearest to the full seven figure value are invariably taken to represent the standard values of x, y, z, we shall for some wave-lengths obtain $x + y + z = 0{\cdot}9999$ and for others $x + y + z = 1{\cdot}0001$. The former occurs when the additional figures for x, y, z each represent a magnitude less than 0·00005 but are not all zero, and the latter when the figures after the fourth place all represent a greater quantity than 0·00005. In the former case the greatest remainder was increased, and in the latter the least remainder was reduced, to secure $r + g + b = 1{\cdot}0000$ for every wave-length. The number of coefficients modified in this way was decidedly less than the proportion for which it would be necessary with an indefinitely large random collection of numbers arranged in threes to sum to unity.

It will be seen that the system of units **X**, **Y**, **Z** has been evolved by making use of the properties of the standard observer defined in Resolution (1). For this observer the equi-energy source is represented by the centroid of the colour triangle XYZ. From the values of the ω's given in (2), we can form the following unit equations for the three spectral colours:

$$\left.\begin{aligned}\mathbf{R} &= 0{\cdot}73467\,\mathbf{X} + 0{\cdot}26533\,\mathbf{Y} + 0{\cdot}00000\,\mathbf{Z}\\ \mathbf{G} &= 0{\cdot}27376\,\mathbf{X} + 0{\cdot}71741\,\mathbf{Y} + 0{\cdot}00883\,\mathbf{Z}\\ \mathbf{B} &= 0{\cdot}16658\,\mathbf{X} + 0{\cdot}00886\,\mathbf{Y} + 0{\cdot}82456\,\mathbf{Z}\end{aligned}\right\} \qquad \ldots\ldots(3).$$

In accordance with Resolution (4) the standard system is now defined by means of these equations. We have still to provide a means of establishing the units for direct colorimetry. In our laboratories we have neither a source emitting the equi-energy spectrum nor a real observer exactly equivalent to the standard observer. We bridge the gap by accepting as correct the energy distribution of radiation from the standard source B found by Davis and Gibson and supplementing it by the interpolates given in Table V. On this basis, following the process already described, it is found that the standard observer would obtain the following colour equation for this source:

$$\mathbf{S}_B = 0{\cdot}34842\,\mathbf{X} + 0{\cdot}35161\,\mathbf{Y} + 0{\cdot}29997\,\mathbf{Z} \qquad \ldots\ldots(4).$$

We adopt these values as part of our definition of the system. These four sets of coordinates provide all that is necessary to enable any experimenter to express his own colour matches on a system which agrees exactly with that of the standard observer for the cardinal stimuli, i.e. the four real stimuli by which the system is defined. The closeness of the agreement elsewhere depends only on the extent to which the colour vision of the experimenter resembles that of the standard observer.

The coordinates of the other two standard sources found in a similar way are given by the equations

$$\mathbf{S}_A = 0{\cdot}44757\,\mathbf{X} + 0{\cdot}40745\,\mathbf{Y} + 0{\cdot}14498\,\mathbf{Z} \qquad \ldots\ldots(5),$$

$$\mathbf{S}_C = 0{\cdot}31006\,\mathbf{X} + 0{\cdot}31616\,\mathbf{Y} + 0{\cdot}37378\,\mathbf{Z} \qquad \ldots\ldots(6).$$

III. EXTENDED SPECTRAL TABLES

Though the colour coordinates given in Table III are properly regarded as a complete definition of the standard observer, it is at times convenient to have at hand for reference values for smaller wave-length intervals. A more detailed table (Table IV) has therefore been prepared giving the coordinates at intervals of 1 mμ. In the preparation of this table Lagrangian interpolation was employed except where it was obviously advantageous to work from differences. The run of the figures is smooth within the limits determined by the use of only four significant figures.

In the computation of this table the seven figure values already mentioned were used, and the interpolations were carried out to the same number of figures in order to obtain a strict check on the accuracy of the calculations. In reducing the values to four figures, adjustments to secure the sum 1·0000 were occasionally required as in the calculations described earlier. For one or two wave-lengths adherence to the strict rule of modifying the largest remainder when the abbreviated sum was 0·9999 and the smallest when it was 1·0001 would have yielded slightly less smooth variations in the coordinates than by modifying a remainder of almost equal value. When this occurred the change giving the smoother result was preferred.

IV. TABLES FOR INDIRECT COLORIMETRY USING THE STANDARD ILLUMINANTS

With the adoption of standard illuminants and a standard observer numerous calculations of colour and brightness factor of materials illuminated by these sources will in future be required. These routine calculations should be simplified to the greatest possible extent by the use of special tables. It is clearly of assistance to have tables in which the products $E_\lambda \bar{x}_\lambda$, $E_\lambda \bar{y}_\lambda$, $E_\lambda \bar{z}_\lambda$ are recorded, so that it is only necessary to multiply these by f_λ, the measured transmission or reflection factor of the material for wave-length λ. The scale on which E_λ is measured is arbitrary, and can be adjusted in each table to secure any property which appears to be of value. As the visibility of the light is determined by the unit **Y**, the brightness of the source is measured by $\int E_\lambda \bar{y}_\lambda d\lambda$, and that of the light received from the material by $\int f_\lambda E_\lambda \bar{y}_\lambda d\lambda$. The brightness factor[*] β of the material is therefore given by

$$\beta = \frac{\int f_\lambda E_\lambda \bar{y}_\lambda d\lambda}{\int E_\lambda \bar{y}_\lambda d\lambda}.$$

If we arrange the scale of E so that the denominator of this fraction has the value 100, the numerator alone will give the percentage brightness of the material. Tables VI, VII, VIII are computed on this basis. The number of figures given is sufficient to meet all needs however restricted the spectral region through which light is transmitted or reflected by the material may be. This implies that the values in some parts of the tables are specified to an accuracy that is meaningless. It does not follow that in calculations these figures should be reduced. The tables depend only on the standard observer and the agreed properties of the standard illuminants, and the use of the figures as they stand ensures that all computers will deduce the same brightness factors and the same colour coordinates from given spectrophotometric measurements.

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[*] This term replaces "shade," used in the instructions issued with the Guild colorimeter. For transparent materials the brightness factor is identical with the photometric transmission. For opaque materials it is interpreted in accordance with Resolutions (3) and (3*a*).

The three colour coordinates x, y, z are of course proportional to

$$\int f_\lambda E_\lambda \bar{x}_\lambda d\lambda, \quad \int f_\lambda E_\lambda \bar{y}_\lambda d\lambda, \quad \int f_\lambda E_\lambda \bar{z}_\lambda d\lambda.$$

The coordinates of the three standard illuminants given in equations (4), (5), and (6) are obtained from the values of these integrals when $f_\lambda = 1$. These values are recorded as totals at the foot of each table.

V. RELATIONS CONNECTING THE N.P.L. AND THE C.I.E. SYSTEMS

Standards specifications which have hitherto been expressed on the N.P.L. system will in future be given on the C.I.E. system. It is therefore necessary to state the relations by which the one can be derived from the other. Occasions will doubtless arise when it is desired to compare a specification on the C.I.E. system with measurements recorded on the N.P.L. system, and it is therefore convenient to have the transformation equations given for use in both directions. For the sake of completeness we give here also the relations connecting the system of Resolution (1) with the other two systems. The units of the N.P.L. system are denoted by R', G', B' and coordinates on this system by r', g', b'; on the Resolution (1) system the corresponding quantities are R, G, B and r, g, b; and on the C.I.E. system we have $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$ and x, y, z. W denotes the basic stimulus of the N.P.L. system—the N.P.L. White Light. The connections between the systems are involved in the coordinates of the stimuli named in the first column of Table IX on the three systems.

The equations connecting the systems may be written

$$\left.\begin{array}{lll} R' = 1{\cdot}27663R, & G' = 1{\cdot}22460G, & B' = B \\ R = 0{\cdot}78331R', & G = 0{\cdot}81659G', & B = B' \end{array}\right\} \begin{array}{l}\text{N.P.L. and} \\ \text{Resolution (1)}\end{array}$$

$$\left.\begin{array}{rcrrr} R' &=& 0{\cdot}62555\mathbf{X} & +0{\cdot}22592\mathbf{Y} & +0{\cdot}00000\mathbf{Z} \\ G' &=& 0{\cdot}37963\mathbf{X} & +0{\cdot}99487\mathbf{Y} & +0{\cdot}0122\underline{5}\mathbf{Z} \\ B' &=& 0{\cdot}20000\mathbf{X} & +0{\cdot}01063\mathbf{Y} & +0{\cdot}99000\mathbf{Z} \\ \mathbf{X} &=& 1{\cdot}85221R' & -0{\cdot}42067G' & +0{\cdot}00520B' \\ \mathbf{Y} &=& -0{\cdot}70226R' & +1{\cdot}16478G' & -0{\cdot}01441B' \\ \mathbf{Z} &=& -0{\cdot}36664R' & +0{\cdot}07247G' & +1{\cdot}00921B' \end{array}\right\} \begin{array}{l}\text{N.P.L. and} \\ \text{C.I.E.}\end{array}$$

$$\left.\begin{array}{rcrrr} R &=& 0{\cdot}49000\mathbf{X} & +0{\cdot}17697\mathbf{Y} & +0{\cdot}00000\mathbf{Z} \\ G &=& 0{\cdot}31000\mathbf{X} & +0{\cdot}81240\mathbf{Y} & +0{\cdot}01000\mathbf{Z} \\ B &=& 0{\cdot}20000\mathbf{X} & +0{\cdot}01063\mathbf{Y} & +0{\cdot}99000\mathbf{Z} \\ \mathbf{X} &=& 2{\cdot}36460R & -0{\cdot}5151\underline{5}G & +0{\cdot}00520B \\ \mathbf{Y} &=& -0{\cdot}89654R & +1{\cdot}42640G & -0{\cdot}01441B \\ \mathbf{Z} &=& -0{\cdot}46807R & +0{\cdot}0887\underline{5}G & +1{\cdot}00921B \end{array}\right\} \begin{array}{l}\text{Resolution (1)} \\ \text{and C.I.E.}\end{array}$$

In practice it is also convenient to have the equations connecting the standard system with three systems having R, G, B as unitary stimuli and S_A, S_B, S_C respectively as basic stimuli. The equations are

$$\left.\begin{aligned}
R_A &= 0\cdot84277\,\mathbf{X} + 0\cdot30437\,\mathbf{Y} + 0\cdot00000\,\mathbf{Z}\\
G_A &= 0\cdot31000\,\mathbf{X} + 0\cdot81240\,\mathbf{Y} + 0\cdot01000\,\mathbf{Z}\\
B_A &= 0\cdot07856\,\mathbf{X} + 0\cdot00418\,\mathbf{Y} + 0\cdot38886\,\mathbf{Z}\\
\mathbf{X} &= 1\cdot37481\,R_A - 0\cdot5151\underline{5}\,G_A + 0\cdot0132\underline{5}\,B_A\\
\mathbf{Y} &= -0\cdot52126\,R_A + 1\cdot42640\,G_A - 0\cdot03668\,B_A\\
\mathbf{Z} &= -0\cdot27214\,R_A + 0\cdot0887\underline{5}\,G_A + 2\cdot56934\,B_A
\end{aligned}\right\}\begin{matrix} R, G, B : S_A \\ \text{and C.I.E.}\end{matrix}$$

$$\left.\begin{aligned}
R_B &= 0\cdot51750\,\mathbf{X} + 0\cdot18690\,\mathbf{Y} + 0\cdot00000\,\mathbf{Z}\\
G_B &= 0\cdot31000\,\mathbf{X} + 0\cdot81240\,\mathbf{Y} + 0\cdot01000\,\mathbf{Z}\\
B_B &= 0\cdot17178\,\mathbf{X} + 0\cdot00913\,\mathbf{Y} + 0\cdot85032\,\mathbf{Z}\\
\mathbf{X} &= 2\cdot23893\,R_B - 0\cdot5151\underline{5}\,G_B + 0\cdot00606\,B_B\\
\mathbf{Y} &= -0\cdot84889\,R_B + 1\cdot42640\,G_B - 0\cdot01677\,B_B\\
\mathbf{Z} &= -0\cdot44319\,R_B + 0\cdot0887\underline{5}\,G_B + 1\cdot17498\,B_B
\end{aligned}\right\}\begin{matrix} R, G, B : S_B \\ \text{and C.I.E.}\end{matrix}$$

$$\left.\begin{aligned}
R_C &= 0\cdot41500\,\mathbf{X} + 0\cdot14988\,\mathbf{Y} + 0\cdot00000\,\mathbf{Z}\\
G_C &= 0\cdot31000\,\mathbf{X} + 0\cdot81240\,\mathbf{Y} + 0\cdot01000\,\mathbf{Z}\\
B_C &= 0\cdot23074\,\mathbf{X} + 0\cdot12268\,\mathbf{Y} + 1\cdot14216\,\mathbf{Z}\\
\mathbf{X} &= 2\cdot79193\,R_C - 0\cdot5151\underline{5}\,G_C + 0\cdot00451\,B_C\\
\mathbf{Y} &= -1\cdot05856\,R_C + 1\cdot42640\,G_C - 0\cdot01249\,B_C\\
\mathbf{Z} &= -0\cdot55266\,R_C + 0\cdot0887\underline{5}\,G_C + 0\cdot87476\,B_C
\end{aligned}\right\}\begin{matrix} R, G, B : S_C \\ \text{and C.I.E.}\end{matrix}$$

In each of these transformations the nine coefficients of the first set of equations form a matrix which will later be denoted by the letter δ, and the second set a matrix denoted by Δ.

Note. When only four decimal places are retained, the figure immediately preceding an underlined 5 is not to be increased.

It should be realised that the number of figures given in these equations does not correspond to the precision attainable in experimental colorimetry, or in the exercise of any other visual faculty. Throughout this paper numbers are computed to an accuracy in excess of practical requirements both to attain unquestionable numerical consistency throughout the system and to ensure that cumulative errors of computation, which might otherwise become significant, will remain negligible in all calculations based on these figures.

Normally time and labour can be saved by using only the number of figures appropriate to the accuracy of the observations or of the results required, both in the transformation equations and in the quantities recorded in the following tables.

Table I. *The* 1931

Trichromatic coefficients			Wave-length in mμ	Distribution coefficients for equal energy stimulus		
r	*g*	*b*		$\bar{r}$	$\bar{g}$	$\bar{b}$
0·0272	− 0·0115	0·9843	**380**	0·00003	− 0·00001	0·00117
0·0268	− 0·0114	0·9846	**385**	0·00005	− 0·00002	0·00189
0·0263	− 0·0114	0·9851	**390**	0·00010	− 0·00004	0·00359
0·0256	− 0·0113	0·9857	**395**	0·00017	− 0·00007	0·00647
0·0247	− 0·0112	0·9865	**400**	0·00030	− 0·00014	0·01214
0·0237	− 0·0111	0·9874	**405**	0·00047	− 0·00022	0·01969
0·0225	− 0·0109	0·9884	**410**	0·00084	− 0·00041	0·03707
0·0207	− 0·0104	0·9897	**415**	0·00139	− 0·00070	0·06637
0·0181	− 0·0094	0·9913	**420**	0·00211	− 0·00110	0·11541
0·0142	− 0·0076	0·9934	**425**	0·00266	− 0·00143	0·18575
0·0088	− 0·0048	0·9960	**430**	0·00218	− 0·00119	0·24769
0·0012	− 0·0007	0·9995	**435**	0·00036	− 0·00021	0·29012
− 0·0084	0·0048	1·0036	**440**	− 0·00261	0·00149	0·31228
− 0·0213	0·0120	1·0093	**445**	− 0·00673	0·00379	0·31860
− 0·0390	0·0218	1·0172	**450**	− 0·01213	0·00678	0·31670
− 0·0618	0·0345	1·0273	**455**	− 0·01874	0·01046	0·31166
− 0·0909	0·0517	1·0392	**460**	− 0·02608	0·01485	0·29821
− 0·1281	0·0762	1·0519	**465**	− 0·03324	0·01977	0·27295
− 0·1821	0·1175	1·0646	**470**	− 0·03933	0·02538	0·22991
− 0·2584	0·1840	1·0744	**475**	− 0·04471	0·03183	0·18592
− 0·3667	0·2906	1·0761	**480**	− 0·04939	0·03914	0·14494
− 0·5200	0·4568	1·0632	**485**	− 0·05364	0·04713	0·10968
− 0·7150	0·6996	1·0154	**490**	− 0·05814	0·05689	0·08257
− 0·9459	1·0247	0·9212	**495**	− 0·06414	0·06948	0·06246
− 1·1685	1·3905	0·7780	**500**	− 0·07173	0·08536	0·04776
− 1·3182	1·7195	0·5987	**505**	− 0·08120	0·10593	0·03688
− 1·3371	1·9318	0·4053	**510**	− 0·08901	0·12860	0·02698
− 1·2076	1·9699	0·2377	**515**	− 0·09356	0·15262	0·01842
− 0·9830	1·8534	0·1296	**520**	− 0·09264	0·17468	0·01221
− 0·7386	1·6662	0·0724	**525**	− 0·08473	0·19113	0·00830
− 0·5159	1·4761	0·0398	**530**	− 0·07101	0·20317	0·00549
− 0·3304	1·3105	0·0199	**535**	− 0·05316	0·21083	0·00320
− 0·1707	1·1628	0·0079	**540**	− 0·03152	0·21466	0·00146
− 0·0293	1·0282	0·0011	**545**	− 0·00613	0·21487	0·00023
0·0974	0·9051	− 0·0025	**550**	0·02279	0·21178	− 0·00058
0·2121	0·7919	− 0·0040	**555**	0·05514	0·20588	− 0·00105
0·3164	0·6881	− 0·0045	**560**	0·09060	0·19702	− 0·00130
0·4112	0·5932	− 0·0044	**565**	0·12840	0·18522	− 0·00138
0·4973	0·5067	− 0·0040	**570**	0·16768	0·17087	− 0·00135
0·5751	0·4283	− 0·0034	**575**	0·20715	0·15429	− 0·00123
0·6449	0·3579	− 0·0028	**580**	0·24526	0·13610	− 0·00108
Column 1	2	3	4	5	6	7

Ratio of luminosity factors

C.I.E. standard observer

Trichromatic coefficients			Wave-length in mμ	Distribution coefficients for equal energy stimulus		
r	g	b		$\bar{r}$	$\bar{g}$	$\bar{b}$
0·6449	0·3579	− 0·0028	**580**	0·24526	0·13610	− 0·00108
0·7071	0·2952	− 0·0023	**585**	0·27989	0·11686	− 0·00093
0·7617	0·2402	− 0·0019	**590**	0·30928	0·09754	− 0·00079
0·8087	0·1928	− 0·0015	**595**	0·33184	0·07909	− 0·00063
0·8475	0·1537	− 0·0012	**600**	0·34429	0·06246	− 0·00049
0·8800	0·1209	− 0·0009	**605**	0·34756	0·04776	− 0·00038
0·9059	0·0949	− 0·0008	**610**	0·33971	0·03557	− 0·00030
0·9265	0·0741	− 0·0006	**615**	0·32265	0·02583	− 0·00022
0·9425	0·0580	− 0·0005	**620**	0·29708	0·01828	− 0·00015
0·9550	0·0454	− 0·0004	**625**	0·26348	0·01253	− 0·00011
0·9649	0·0354	− 0·0003	**630**	0·22677	0·00833	− 0·00008
0·9730	0·0272	− 0·0002	**635**	0·19233	0·00537	− 0·00005
0·9797	0·0205	− 0·0002	**640**	0·15968	0·00334	− 0·00003
0·9850	0·0152	− 0·0002	**645**	0·12905	0·00199	− 0·00002
0·9888	0·0113	− 0·0001	**650**	0·10167	0·00116	− 0·00001
0·9918	0·0083	− 0·0001	**655**	0·07857	0·00066	− 0·00001
0·9940	0·0061	− 0·0001	**660**	0·05932	0·00037	0·00000
0·9954	0·0047	− 0·0001	**665**	0·04366	0·00021	0·00000
0·9966	0·0035	− 0·0001	**670**	0·03149	0·00011	0·00000
0·9975	0·0025	0·0000	**675**	0·02294	0·00006	0·00000
0·9984	0·0016	0·0000	**680**	0·01687	0·00003	0·00000
0·9991	0·0009	0·0000	**685**	0·01187	0·00001	0·00000
0·9996	0·0004	0·0000	**690**	0·00819	0·00000	0·00000
0·9999	0·0001	0·0000	**695**	0·00572	0·00000	0·00000
1·0000	0·0000	0·0000	**700**	0·00410	0·00000	0·00000
1·0000	0·0000	0·0000	**705**	0·00291	0·00000	0·00000
1·0000	0·0000	0·0000	**710**	0·00210	0·00000	0·00000
1·0000	0·0000	0·0000	**715**	0·00148	0·00000	0·00000
1·0000	0·0000	0·0000	**720**	0·00105	0·00000	0·00000
1·0000	0·0000	0·0000	**725**	0·00074	0·00000	0·00000
1·0000	0·0000	0·0000	**730**	0·00052	0·00000	0·00000
1·0000	0·0000	0·0000	**735**	0·00036	0·00000	0·00000
1·0000	0·0000	0·0000	**740**	0·00025	0·00000	0·00000
1·0000	0·0000	0·0000	**745**	0·00017	0·00000	0·00000
1·0000	0·0000	0·0000	**750**	0·00012	0·00000	0·00000
1·0000	0·0000	0·0000	**755**	0·00008	0·00000	0·00000
1·0000	0·0000	0·0000	**760**	0·00006	0·00000	0·00000
1·0000	0·0000	0·0000	**765**	0·00004	0·00000	0·00000
1·0000	0·0000	0·0000	**770**	0·00003	0·00000	0·00000
1·0000	0·0000	0·0000	**775**	0·00001	0·00000	0·00000
1·0000	0·0000	0·0000	**780**	0·00000	0·00000	0·00000
1	2	3	4	5	6	7

$L_R : L_G : L_B = 1 : 4{\cdot}5907 : 0{\cdot}0601.$

Table III. *The* 1931

Trichromatic coefficients			Wave-length in mμ	Distribution coefficients for equal energy stimulus		
x	y	z		$\bar{x}$	$\bar{y}$	$\bar{z}$
0·1741	0·0050	0·8209	**380**	0·0014	0·0000	0·0065
0·1740	0·0050	0·8210	**385**	0·0022	0·0001	0·0105
0·1738	0·0049	0·8213	**390**	0·0042	0·0001	0·0201
0·1736	0·0049	0·8215	**395**	0·0076	0·0002	0·0362
0·1733	0·0048	0·8219	**400**	0·0143	0·0004	0·0679
0·1730	0·0048	0·8222	**405**	0·0232	0·0006	0·1102
0·1726	0·0048	0·8226	**410**	0·0435	0·0012	0·2074
0·1721	0·0048	0·8231	**415**	0·0776	0·0022	0·3713
0·1714	0·0051	0·8235	**420**	0·1344	0·0040	0·6456
0·1703	0·0058	0·8239	**425**	0·2148	0·0073	1·0391
0·1689	0·0069	0·8242	**430**	0·2839	0·0116	1·3856
0·1669	0·0086	0·8245	**435**	0·3285	0·0168	1·6230
0·1644	0·0109	0·8247	**440**	0·3483	0·0230	1·7471
0·1611	0·0138	0·8251	**445**	0·3481	0·0298	1·7826
0·1566	0·0177	0·8257	**450**	0·3362	0·0380	1·7721
0·1510	0·0227	0·8263	**455**	0·3187	0·0480	1·7441
0·1440	0·0297	0·8263	**460**	0·2908	0·0600	1·6692
0·1355	0·0399	0·8246	**465**	0·2511	0·0739	1·5281
0·1241	0·0578	0·8181	**470**	0·1954	0·0910	1·2876
0·1096	0·0868	0·8036	**475**	0·1421	0·1126	1·0419
0·0913	0·1327	0·7760	**480**	0·0956	0·1390	0·8130
0·0687	0·2007	0·7306	**485**	0·0580	0·1693	0·6162
0·0454	0·2950	0·6596	**490**	0·0320	0·2080	0·4652
0·0235	0·4127	0·5638	**495**	0·0147	0·2586	0·3533
0·0082	0·5384	0·4534	**500**	0·0049	0·3230	0·2720
0·0039	0·6548	0·3413	**505**	0·0024	0·4073	0·2123
0·0139	0·7502	0·2359	**510**	0·0093	0·5030	0·1582
0·0389	0·8120	0·1491	**515**	0·0291	0·6082	0·1117
0·0743	0·8338	0·0919	**520**	0·0633	0·7100	0·0782
0·1142	0·8262	0·0596	**525**	0·1096	0·7932	0·0573
0·1547	0·8059	0·0394	**530**	0·1655	0·8620	0·0422
0·1929	0·7816	0·0255	**535**	0·2257	0·9149	0·0298
0·2296	0·7543	0·0161	**540**	0·2904	0·9540	0·0203
0·2658	0·7243	0·0099	**545**	0·3597	0·9803	0·0134
0·3016	0·6923	0·0061	**550**	0·4334	0·9950	0·0087
0·3373	0·6589	0·0038	**555**	0·5121	1·0002	0·0057
0·3731	0·6245	0·0024	**560**	0·5945	0·9950	0·0039
0·4087	0·5896	0·0017	**565**	0·6784	0·9786	0·0027
0·4441	0·5547	0·0012	**570**	0·7621	0·9520	0·0021
0·4788	0·5202	0·0010	**575**	0·8425	0·9154	0·0018
0·5125	0·4866	0·0009	**580**	0·9163	0·8700	0·0017
Column 1	2	3	4	5	6	7

The luminosity factors are zero,

C.I.E. standard observer

Trichromatic coefficients			Wave-length in mμ	Distribution coefficients for equal energy stimulus		
x	*y*	*z*		$\bar{x}$	$\bar{y}$	$\bar{z}$
0·5125	0·4866	0·0009	**580**	0·9163	0·8700	0·0017
0·5448	0·4544	0·0008	**585**	0·9786	0·8163	0·0014
0·5752	0·4242	0·0006	**590**	1·0263	0·7570	0·0011
0·6029	0·3965	0·0006	**595**	1·0567	0·6949	0·0010
0·6270	0·3725	0·0005	**600**	1·0622	0·6310	0·0008
0·6482	0·3514	0·0004	**605**	1·0456	0·5668	0·0006
0·6658	0·3340	0·0002	**610**	1·0026	0·5030	0·0003
0·6801	0·3197	0·0002	**615**	0·9384	0·4412	0·0002
0·6915	0·3083	0·0002	**620**	0·8544	0·3810	0·0002
0·7006	0·2993	0·0001	**625**	0·7514	0·3210	0·0001
0·7079	0·2920	0·0001	**630**	0·6424	0·2650	0·0000
0·7140	0·2859	0·0001	**635**	0·5419	0·2170	0·0000
0·7190	0·2809	0·0001	**640**	0·4479	0·1750	0·0000
0·7230	0·2770	0·0000	**645**	0·3608	0·1382	0·0000
0·7260	0·2740	0·0000	**650**	0·2835	0·1070	0·0000
0·7283	0·2717	0·0000	**655**	0·2187	0·0816	0·0000
0·7300	0·2700	0·0000	**660**	0·1649	0·0610	0·0000
0·7311	0·2689	0·0000	**665**	0·1212	0·0446	0·0000
0·7320	0·2680	0·0000	**670**	0·0874	0·0320	0·0000
0·7327	0·2673	0·0000	**675**	0·0636	0·0232	0·0000
0·7334	0·2666	0·0000	**680**	0·0468	0·0170	0·0000
0·7340	0·2660	0·0000	**685**	0·0329	0·0119	0·0000
0·7344	0·2656	0·0000	**690**	0·0227	0·0082	0·0000
0·7346	0·2654	0·0000	**695**	0·0158	0·0057	0·0000
0·7347	0·2653	0·0000	**700**	0·0114	0·0041	0·0000
0·7347	0·2653	0·0000	**705**	0·0081	0·0029	0·0000
0·7347	0·2653	0·0000	**710**	0·0058	0·0021	0·0000
0·7347	0·2653	0·0000	**715**	0·0041	0·0015	0·0000
0·7347	0·2653	0·0000	**720**	0·0029	0·0010	0·0000
0·7347	0·2653	0·0000	**725**	0·0020	0·0007	0·0000
0·7347	0·2653	0·0000	**730**	0·0014	0·0005	0·0000
0·7347	0·2653	0·0000	**735**	0·0010	0·0004	0·0000
0·7347	0·2653	0·0000	**740**	0·0007	0·0003	0·0000
0·7347	0·2653	0·0000	**745**	0·0005	0·0002	0·0000
0·7347	0·2653	0·0000	**750**	0·0003	0·0001	0·0000
0·7347	0·2653	0·0000	**755**	0·0002	0·0001	0·0000
0·7347	0·2653	0·0000	**760**	0·0002	0·0001	0·0000
0·7347	0·2653	0·0000	**765**	0·0001	0·0000	0·0000
0·7347	0·2653	0·0000	**770**	0·0001	0·0000	0·0000
0·7347	0·2653	0·0000	**775**	0·0000	0·0000	0·0000
0·7347	0·2653	0·0000	**780**	0·0000	0·0000	0·0000
			Totals	21·3713	21·3714	21·3715
1	2	3	4	5	6	7

unity, and zero respectively.

Table IV. *Spectral colour*

Wave-length in mμ	*x*	*y*	*z*	Wave-length in mμ	*x*	*y*	*z*
380	0·1741	0·0050	0·8209	**420**	0·1714	0·0051	0·8235
381	0·1741	0·0050	0·8209	**421**	0·1712	0·0052	0·8236
382	0·1741	0·0050	0·8209	**422**	0·1710	0·0053	0·8237
383	0·1740	0·0050	0·8210	**423**	0·1708	0·0055	0·8237
384	0·1740	0·0050	0·8210	**424**	0·1706	0·0056	0·8238
385	0·1740	0·0050	0·8210	**425**	0·1703	0·0058	0·8239
386	0·1739	0·0050	0·8211	**426**	0·1700	0·0060	0·8240
387	0·1739	0·0050	0·8211	**427**	0·1698	0·0062	0·8240
388	0·1739	0·0049	0·8212	**428**	0·1695	0·0064	0·8241
389	0·1739	0·0049	0·8212	**429**	0·1692	0·0066	0·8242
390	0·1738	0·0049	0·8213	**430**	0·1689	0·0069	0·8242
391	0·1738	0·0049	0·8213	**431**	0·1685	0·0072	0·8243
392	0·1737	0·0049	0·8214	**432**	0·1681	0·0075	0·8244
393	0·1737	0·0049	0·8214	**433**	0·1678	0·0078	0·8244
394	0·1736	0·0049	0·8215	**434**	0·1673	0·0082	0·8245
395	0·1736	0·0049	0·8215	**435**	0·1669	0·0086	0·8245
396	0·1735	0·0049	0·8216	**436**	0·1664	0·0090	0·8246
397	0·1735	0·0049	0·8216	**437**	0·1660	0·0094	0·8246
398	0·1734	0·0049	0·8217	**438**	0·1655	0·0099	0·8246
399	0·1734	0·0048	0·8218	**439**	0·1650	0·0103	0·8247
400	0·1733	0·0048	0·8219	**440**	0·1644	0·0109	0·8247
401	0·1733	0·0048	0·8219	**441**	0·1638	0·0114	0·8248
402	0·1732	0·0048	0·8220	**442**	0·1632	0·0120	0·8248
403	0·1731	0·0048	0·8221	**443**	0·1626	0·0125	0·8249
404	0·1731	0·0048	0·8221	**444**	0·1619	0·0131	0·8250
405	0·1730	0·0048	0·8222	**445**	0·1611	0·0138	0·8251
406	0·1729	0·0048	0·8223	**446**	0·1603	0·0145	0·8252
407	0·1728	0·0048	0·8224	**447**	0·1595	0·0152	0·8253
408	0·1728	0·0048	0·8224	**448**	0·1586	0·0160	0·8254
409	0·1727	0·0048	0·8225	**449**	0·1576	0·0169	0·8255
410	0·1726	0·0048	0·8226	**450**	0·1566	0·0177	0·8257
411	0·1725	0·0048	0·8227	**451**	0·1556	0·0186	0·8258
412	0·1724	0·0048	0·8228	**452**	0·1545	0·0196	0·8259
413	0·1723	0·0048	0·8229	**453**	0·1534	0·0205	0·8261
414	0·1722	0·0048	0·8230	**454**	0·1522	0·0216	0·8262
415	0·1721	0·0048	0·8231	**455**	0·1510	0·0227	0·8263
416	0·1720	0·0049	0·8231	**456**	0·1497	0·0239	0·8264
417	0·1719	0·0049	0·8232	**457**	0·1484	0·0252	0·8264
418	0·1717	0·0050	0·8233	**458**	0·1469	0·0266	0·8265
419	0·1716	0·0050	0·8234	**459**	0·1455	0·0281	0·8264
420	0·1714	0·0051	0·8235	**460**	0·1440	0·0297	0·8263

coordinates at intervals of 1 mμ

Wave-length in mμ	*x*	*y*	*z*	Wave-length in mμ	*x*	*y*	*z*
460	0·1440	0·0297	0·8263	**500**	0·0082	0·5384	0·4534
461	0·1425	0·0313	0·8262	**501**	0·0063	0·5628	0·4309
462	0·1409	0·0331	0·8260	**502**	0·0049	0·5868	0·4083
463	0·1392	0·0351	0·8257	**503**	0·0040	0·6102	0·3858
464	0·1374	0·0374	0·8252	**504**	0·0037	0·6328	0·3635
465	0·1355	0·0399	0·8246	**505**	0·0039	0·6548	0·3413
466	0·1335	0·0427	0·8238	**506**	0·0047	0·6760	0·3193
467	0·1313	0·0459	0·8228	**507**	0·0061	0·6962	0·2977
468	0·1290	0·0495	0·8215	**508**	0·0081	0·7154	0·2765
469	0·1267	0·0534	0·8199	**509**	0·0107	0·7334	0·2559
470	0·1241	0·0578	0·8181	**510**	0·0139	0·7502	0·2359
471	0·1215	0·0626	0·8159	**511**	0·0178	0·7655	0·2167
472	0·1187	0·0678	0·8135	**512**	0·0223	0·7793	0·1984
473	0·1158	0·0735	0·8107	**513**	0·0274	0·7917	0·1809
474	0·1128	0·0798	0·8074	**514**	0·0329	0·8027	0·1644
475	0·1096	0·0868	0·8036	**515**	0·0389	0·8120	0·1491
476	0·1062	0·0945	0·7993	**516**	0·0453	0·8194	0·1353
477	0·1027	0·1029	0·7944	**517**	0·0521	0·8250	0·1229
478	0·0991	0·1120	0·7889	**518**	0·0593	0·8292	0·1115
479	0·0953	0·1219	0·7828	**519**	0·0667	0·8322	0·1011
480	0·0913	0·1327	0·7760	**520**	0·0743	0·8338	0·0919
481	0·0870	0·1444	0·7686	**521**	0·0821	0·8341	0·0838
482	0·0826	0·1570	0·7604	**522**	0·0899	0·8334	0·0767
483	0·0780	0·1706	0·7514	**523**	0·0979	0·8317	0·0704
484	0·0734	0·1851	0·7415	**524**	0·1060	0·8292	0·0648
485	0·0687	0·2007	0·7306	**525**	0·1142	0·8262	0·0596
486	0·0640	0·2175	0·7185	**526**	0·1223	0·8228	0·0549
487	0·0594	0·2354	0·7052	**527**	0·1305	0·8191	0·0504
488	0·0547	0·2543	0·6910	**528**	0·1386	0·8150	0·0464
489	0·0500	0·2742	0·6758	**529**	0·1467	0·8105	0·0428
490	0·0454	0·2950	0·6596	**530**	0·1547	0·8059	0·0394
491	0·0407	0·3170	0·6423	**531**	0·1625	0·8013	0·0362
492	0·0361	0·3401	0·6238	**532**	0·1702	0·7966	0·0332
493	0·0317	0·3638	0·6045	**533**	0·1778	0·7918	0·0304
494	0·0275	0·3880	0·5845	**534**	0·1854	0·7868	0·0278
495	0·0235	0·4127	0·5638	**535**	0·1929	0·7816	0·0255
496	0·0198	0·4378	0·5424	**536**	0·2003	0·7764	0·0233
497	0·0163	0·4630	0·5207	**537**	0·2077	0·7711	0·0212
498	0·0132	0·4883	0·4985	**538**	0·2151	0·7656	0·0193
499	0·0105	0·5134	0·4761	**539**	0·2224	0·7600	0·0176
500	0·0082	0·5384	0·4534	**540**	0·2296	0·7543	0·0161

Table IV (*cont.*). *Spectral colour*

Wave-length in mμ	*x*	*y*	*z*	Wave-length in mμ	*x*	*y*	*z*
540	0·2296	0·7543	0·0161	**580**	0·5125	0·4866	0·0009
541	0·2369	0·7485	0·0146	**581**	0·5191	0·4800	0·0009
542	0·2441	0·7426	0·0133	**582**	0·5256	0·4735	0·0009
543	0·2514	0·7366	0·0120	**583**	0·5321	0·4671	0·0008
544	0·2586	0·7305	0·0109	**584**	0·5385	0·4607	0·0008
545	0·2658	0·7243	0·0099	**585**	0·5448	0·4544	0·0008
546	0·2729	0·7181	0·0090	**586**	0·5510	0·4482	0·0008
547	0·2801	0·7118	0·0081	**587**	0·5572	0·4421	0·0007
548	0·2873	0·7054	0·0073	**588**	0·5633	0·4360	0·0007
549	0·2944	0·6989	0·0067	**589**	0·5692	0·4301	0·0007
550	0·3016	0·6923	0·0061	**590**	0·5752	0·4242	0·0006
551	0·3088	0·6857	0·0055	**591**	0·5810	0·4184	0·0006
552	0·3159	0·6791	0·0050	**592**	0·5866	0·4128	0·0006
553	0·3231	0·6724	0·0045	**593**	0·5922	0·4072	0·0006
554	0·3302	0·6657	0·0041	**594**	0·5976	0·4018	0·0006
555	0·3373	0·6589	0·0038	**595**	0·6029	0·3965	0·0006
556	0·3445	0·6520	0·0035	**596**	0·6080	0·3914	0·0006
557	0·3517	0·6451	0·0032	**597**	0·6130	0·3865	0·0005
558	0·3588	0·6383	0·0029	**598**	0·6178	0·3817	0·0005
559	0·3660	0·6314	0·0026	**599**	0·6225	0·3770	0·0005
560	0·3731	0·6245	0·0024	**600**	0·6270	0·3725	0·0005
561	0·3802	0·6175	0·0023	**601**	0·6315	0·3680	0·0005
562	0·3874	0·6105	0·0021	**602**	0·6359	0·3637	0·0004
563	0·3945	0·6036	0·0019	**603**	0·6402	0·3594	0·0004
564	0·4016	0·5966	0·0018	**604**	0·6443	0·3553	0·0004
565	0·4087	0·5896	0·0017	**605**	0·6482	0·3514	0·0004
566	0·4158	0·5826	0·0016	**606**	0·6520	0·3477	0·0003
567	0·4229	0·5756	0·0015	**607**	0·6557	0·3440	0·0003
568	0·4300	0·5686	0·0014	**608**	0·6592	0·3405	0·0003
569	0·4371	0·5616	0·0013	**609**	0·6625	0·3372	0·0003
570	0·4441	0·5547	0·0012	**610**	0·6658	0·3340	0·0002
571	0·4510	0·5478	0·0012	**611**	0·6689	0·3309	0·0002
572	0·4580	0·5409	0·0011	**612**	0·6719	0·3279	0·0002
573	0·4650	0·5339	0·0011	**613**	0·6747	0·3251	0·0002
574	0·4719	0·5270	0·0011	**614**	0·6774	0·3224	0·0002
575	0·4788	0·5202	0·0010	**615**	0·6801	0·3197	0·0002
576	0·4856	0·5134	0·0010	**616**	0·6826	0·3172	0·0002
577	0·4924	0·5066	0·0010	**617**	0·6849	0·3149	0·0002
578	0·4991	0·4999	0·0010	**618**	0·6872	0·3126	0·0002
579	0·5058	0·4932	0·0010	**619**	0·6894	0·3104	0·0002
580	0·5125	0·4866	0·0009	**620**	0·6915	0·3083	0·0002

coordinates at intervals of 1mμ

Wave-length in mμ	*x*	*y*	*z*	Wave-length in mμ	*x*	*y*	*z*
620	0·6915	0·3083	0·0002	**660**	0·7300	0·2700	0·0000
621	0·6935	0·3064	0·0001	**661**	0·7302	0·2698	0·0000
622	0·6954	0·3045	0·0001	**662**	0·7305	0·2695	0·0000
623	0·6972	0·3027	0·0001	**663**	0·7307	0·2693	0·0000
624	0·6989	0·3010	0·0001	**664**	0·7309	0·2691	0·0000
625	0·7006	0·2993	0·0001	**665**	0·7311	0·2689	0·0000
626	0·7022	0·2977	0·0001	**666**	0·7313	0·2687	0·0000
627	0·7037	0·2962	0·0001	**667**	0·7315	0·2685	0·0000
628	0·7051	0·2948	0·0001	**668**	0·7317	0·2683	0·0000
629	0·7066	0·2933	0·0001	**669**	0·7318	0·2682	0·0000
630	0·7079	0·2920	0·0001	**670**	0·7320	0·2680	0·0000
631	0·7092	0·2907	0·0001	**671**	0·7321	0·2679	0·0000
632	0·7105	0·2894	0·0001	**672**	0·7323	0·2677	0·0000
633	0·7117	0·2882	0·0001	**673**	0·7324	0·2676	0·0000
634	0·7129	0·2870	0·0001	**674**	0·7326	0·2674	0·0000
635	0·7140	0·2859	0·0001	**675**	0·7327	0·2673	0·0000
636	0·7151	0·2848	0·0001	**676**	0·7329	0·2671	0·0000
637	0·7161	0·2838	0·0001	**677**	0·7330	0·2670	0·0000
638	0·7171	0·2828	0·0001	**678**	0·7331	0·2669	0·0000
639	0·7181	0·2818	0·0001	**679**	0·7333	0·2667	0·0000
640	0·7190	0·2809	0·0001	**680**	0·7334	0·2666	0·0000
641	0·7199	0·2800	0·0001	**681**	0·7335	0·2665	0·0000
642	0·7208	0·2792	0·0000	**682**	0·7337	0·2663	0·0000
643	0·7216	0·2784	0·0000	**683**	0·7338	0·2662	0·0000
644	0·7223	0·2777	0·0000	**684**	0·7339	0·2661	0·0000
645	0·7230	0·2770	0·0000	**685**	0·7340	0·2660	0·0000
646	0·7237	0·2763	0·0000	**686**	0·7341	0·2659	0·0000
647	0·7243	0·2757	0·0000	**687**	0·7342	0·2658	0·0000
648	0·7249	0·2751	0·0000	**688**	0·7342	0·2658	0·0000
649	0·7255	0·2745	0·0000	**689**	0·7343	0·2657	0·0000
650	0·7260	0·2740	0·0000	**690**	0·7344	0·2656	0·0000
651	0·7265	0·2735	0·0000	**691**	0·7344	0·2656	0·0000
652	0·7270	0·2730	0·0000	**692**	0·7345	0·2655	0·0000
653	0·7274	0·2726	0·0000	**693**	0·7345	0·2655	0·0000
654	0·7279	0·2721	0·0000	**694**	0·7346	0·2654	0·0000
655	0·7283	0·2717	0·0000	**695**	0·7346	0·2654	0·0000
656	0·7287	0·2713	0·0000	**696**	0·7346	0·2654	0·0000
657	0·7290	0·2710	0·0000	**697**	0·7346	0·2654	0·0000
658	0·7294	0·2706	0·0000	**698**	0·7347	0·2653	0·0000
659	0·7297	0·2703	0·0000	**699**	0·7347	0·2653	0·0000
660	0·7300	0·2700	0·0000	**700**	0·7347	0·2653	0·0000

Table V. *Energy distribution of the three standard illuminants A, B, C for colorimetry*

Wave-length in mμ	E_A	E_B	E_C	Wave-length in mμ	E_A	E_B	E_C
380	9·79	22·40	33·00	**580**	114·44	101·00	97·80
385	10·90	26·85	39·92	**585**	118·08	100·07	95·43
390	12·09	31·30	47·40	**590**	121·73	99·20	93·20
395	13·36	36·18	55·17	**595**	125·39	98·44	91·22
400	14·71	41·30	63·30	**600**	129·04	98·00	89·70
405	16·15	46·62	71·81	**605**	132·70	98·08	88·83
410	17·68	52·10	80·60	**610**	136·34	98·50	88·40
415	19·29	57·70	89·53	**615**	139·99	99·06	88·19
420	21·00	63·20	98·10	**620**	143·62	99·70	88·10
425	22·79	68·37	105·80	**625**	147·23	100·36	88·06
430	24·67	73·10	112·40	**630**	150·83	101·00	88·00
435	26·64	77·31	117·75	**635**	154·42	101·56	87·86
440	28·70	80·80	121·50	**640**	157·98	102·20	87·80
445	30·85	83·44	123·45	**645**	161·51	103·05	87·99
450	33·09	85·40	124·00	**650**	165·03	103·90	88·20
455	35·41	86·88	123·60	**655**	168·51	104·59	88·20
460	37·82	88·30	123·10	**660**	171·96	105·00	87·90
465	40·30	90·08	123·30	**665**	175·38	105·08	87·22
470	42·87	92·00	123·80	**670**	178·77	104·90	86·30
475	45·52	93·75	124·09	**675**	182·12	104·55	85·30
480	48·25	95·20	123·90	**680**	185·43	103·90	84·00
485	51·04	96·23	122·92	**685**	188·70	102·84	82·21
490	53·91	96·50	120·70	**690**	191·93	101·60	80·20
495	56·85	95·71	116·90	**695**	195·12	100·38	78·24
500	59·86	94·20	112·10	**700**	198·26	99·10	76·30
505	62·93	92·37	106·98	**705**	201·36	97·70	74·36
510	66·06	90·70	102·30	**710**	204·41	96·20	72·40
515	69·25	89·65	98·81	**715**	207·41	94·60	70·40
520	72·50	89·50	96·90	**720**	210·36	92·90	68·30
525	75·79	90·43	96·78	**725**	213·26	91·10	66·30
530	79·13	92·20	98·00	**730**	216·12	89·40	64·40
535	82·52	94·46	99·94	**735**	218·92	88·00	62·80
540	85·95	96·90	102·10	**740**	221·66	86·90	61·50
545	89·41	99·16	103·95	**745**	224·36	85·90	60·20
550	92·91	101·00	105·20	**750**	227·00	85·20	59·20
555	96·44	102·20	105·67	**755**	229·58	84·80	58·50
560	100·00	102·80	105·30	**760**	232·11	84·70	58·10
565	103·58	102·92	104·11	**765**	234·59	84·90	58·00
570	107·18	102·60	102·30	**770**	237·01	85·40	58·20
575	110·80	101·90	100·15	**775**	239·37	86·10	58·50
580	114·44	101·00	97·80	**780**	241·67	87·00	59·10

Table VI. *Computational table for standard illuminant A*

Wave-length in mμ	$E\bar{x}$	$E\bar{y}$	$E\bar{z}$	Wave-length in mμ	$E\bar{x}$	$E\bar{y}$	$E\bar{z}$
380	0·0006	0·0000	0·0029	580	4·8594	4·6139	0·0090
385	0·0011	0·0000	0·0053	585	5·3549	4·4668	0·0077
390	0·0024	0·0000	0·0113	590	5·7896	4·2704	0·0062
395	0·0047	0·0001	0·0224	595	6·1403	4·0379	0·0058
400	0·0097	0·0003	0·0463	600	6·3518	3·7733	0·0048
405	0·0174	0·0004	0·0825	605	6·4299	3·4855	0·0037
410	0·0356	0·0010	0·1699	610	6·3346	3·1780	0·0019
415	0·0694	0·0020	0·3319	615	6·0877	2·8622	0·0013
420	0·1308	0·0039	0·6283	620	5·6865	2·5358	0·0013
425	0·2269	0·0077	1·0974	625	5·1267	2·1901	0·0007
430	0·3246	0·0133	1·5840	630	4·4902	1·8523	0·0000
435	0·4055	0·0207	2·0036	635	3·8779	1·5529	0·0000
440	0·4632	0·0306	2·3236	640	3·2791	1·2812	0·0000
445	0·4976	0·0426	2·5484	645	2·7004	1·0344	0·0000
450	0·5155	0·0583	2·7173	650	2·1681	0·8183	0·0000
455	0·5230	0·0788	2·8621	655	1·7078	0·6372	0·0000
460	0·5097	0·1052	2·9254	660	1·3141	0·4861	0·0000
465	0·4690	0·1380	2·8539	665	0·9850	0·3625	0·0000
470	0·3882	0·1808	2·5581	670	0·7241	0·2651	0·0000
475	0·2998	0·2375	2·1979	675	0·5368	0·1958	0·0000
480	0·2138	0·3108	1·8179	680	0·4022	0·1461	0·0000
485	0·1372	0·4004	1·4575	685	0·2877	0·1041	0·0000
490	0·0799	0·5196	1·1622	690	0·2019	0·0729	0·0000
495	0·0387	0·6813	0·9308	695	0·1429	0·0515	0·0000
500	0·0136	0·8960	0·7545	700	0·1047	0·0377	0·0000
505	0·0070	1·1878	0·6191	705	0·0756	0·0271	0·0000
510	0·0285	1·5398	0·4843	710	0·0549	0·0199	0·0000
515	0·0934	1·9518	0·3585	715	0·0394	0·0144	0·0000
520	0·2127	2·3855	0·2627	720	0·0283	0·0097	0·0000
525	0·3849	2·7859	0·2012	725	0·0198	0·0069	0·0000
530	0·6069	3·1609	0·1547	730	0·0140	0·0050	0·0000
535	0·8631	3·4987	0·1140	735	0·0101	0·0041	0·0000
540	1·1567	3·7999	0·0809	740	0·0072	0·0031	0·0000
545	1·4904	4·0618	0·0555	745	0·0052	0·0021	0·0000
550	1·8660	4·2841	0·0375	750	0·0032	0·0010	0·0000
555	2·2887	4·4701	0·0255	755	0·0021	0·0010	0·0000
560	2·7550	4·6110	0·0181	760	0·0021	0·0010	0·0000
565	3·2564	4·6974	0·0130	765	0·0011	0·0000	0·0000
570	3·7853	4·7285	0·0104	770	0·0011	0·0000	0·0000
575	4·3259	4·7002	0·0092	775	0·0000	0·0000	0·0000
580	4·8594	4·6139	0·0090	780	0·0000	0·0000	0·0000
				Totals	109·8472	100·0000	35·5824

Table VII. *Computational table for standard illuminant B*

Wave-length in mμ	$E\bar{x}$	$E\bar{y}$	$E\bar{z}$	Wave-length in mμ	$E\bar{x}$	$E\bar{y}$	$E\bar{z}$
380	0·0015	0·0000	0·0070	**580**	4·4218	4·1984	0·0082
385	0·0028	0·0001	0·0135	**585**	4·6790	3·9030	0·0067
390	0·0063	0·0001	0·0301	**590**	4·8644	3·5880	0·0052
395	0·0131	0·0003	0·0626	**595**	4·9701	3·2684	0·0047
400	0·0282	0·0008	0·1340	**600**	4·9736	2·9546	0·0037
405	0·0517	0·0013	0·2455	**605**	4·8999	2·6561	0·0028
410	0·1083	0·0030	0·5163	**610**	4·7185	2·3672	0·0014
415	0·2139	0·0061	1·0236	**615**	4·4415	2·0882	0·0009
420	0·4058	0·0121	1·9495	**620**	4·0700	1·8149	0·0009
425	0·7017	0·0238	3·3944	**625**	3·6031	1·5392	0·0005
430	0·9916	0·0405	4·8394	**630**	3·1000	1·2788	0·0000
435	1·2134	0·0621	5·9951	**635**	2·6296	1·0530	0·0000
440	1·3446	0·0888	6·7448	**640**	2·1871	0·8545	0·0000
445	1·3878	0·1188	7·1067	**645**	1·7765	0·6804	0·0000
450	1·3718	0·1551	7·2308	**650**	1·4074	0·5312	0·0000
455	1·3229	0·1993	7·2399	**655**	1·0929	0·4078	0·0000
460	1·2269	0·2531	7·0422	**660**	0·8273	0·3060	0·0000
465	1·0807	0·3181	6·5769	**665**	0·6085	0·2239	0·0000
470	0·8589	0·4000	5·6599	**670**	0·4381	0·1604	0·0000
475	0·6365	0·5044	4·6670	**675**	0·3177	0·1159	0·0000
480	0·4348	0·6323	3·6980	**680**	0·2323	0·0844	0·0000
485	0·2667	0·7784	2·8332	**685**	0·1617	0·0585	0·0000
490	0·1475	0·9590	2·1449	**690**	0·1102	0·0398	0·0000
495	0·0672	1·1826	1·6156	**695**	0·0758	0·0273	0·0000
500	0·0221	1·4538	1·2242	**700**	0·0540	0·0194	0·0000
505	0·0106	1·7976	0·9370	**705**	0·0378	0·0135	0·0000
510	0·0403	2·1798	0·6856	**710**	0·0267	0·0097	0·0000
515	0·1246	2·6052	0·4785	**715**	0·0185	0·0068	0·0000
520	0·2707	3·0361	0·3344	**720**	0·0129	0·0044	0·0000
525	0·4735	3·4272	0·2476	**725**	0·0087	0·0030	0·0000
530	0·7291	3·7973	0·1859	**730**	0·0060	0·0021	0·0000
535	1·0186	4·1292	0·1345	**735**	0·0042	0·0017	0·0000
540	1·3445	4·4168	0·0940	**740**	0·0029	0·0012	0·0000
545	1·7042	4·6445	0·0635	**745**	0·0020	0·0008	0·0000
550	2·0915	4·8016	0·0420	**750**	0·0012	0·0004	0·0000
555	2·5006	4·8840	0·0278	**755**	0·0008	0·0004	0·0000
560	2·9200	4·8872	0·0192	**760**	0·0008	0·0004	0·0000
565	3·3360	4·8122	0·0133	**765**	0·0004	0·0000	0·0000
570	3·7359	4·6669	0·0103	**770**	0·0004	0·0000	0·0000
575	4·1019	4·4568	0·0088	**775**	0·0000	0·0000	0·0000
580	4·4218	4·1984	0·0082	**780**	0·0000	0·0000	0·0000
				Totals	99·0930	100·0000	85·3125

Table VIII. *Computational table for standard illuminant C*

Wave-length in mμ	$E\bar{x}$	$E\bar{y}$	$E\bar{z}$	Wave-length in mμ	$E\bar{x}$	$E\bar{y}$	$E\bar{z}$
380	0·0022	0·0000	0·0101	**580**	4·2084	3·9958	0·0078
385	0·0041	0·0002	0·0197	**585**	4·3859	3·6585	0·0063
390	0·0093	0·0002	0·0447	**590**	4·4920	3·3133	0·0048
395	0·0197	0·0005	0·0938	**595**	4·5265	2·9767	0·0043
400	0·0425	0·0012	0·2018	**600**	4·4745	2·6581	0·0034
405	0·0782	0·0020	0·3716	**605**	4·3617	2·3644	0·0025
410	0·1647	0·0045	0·7850	**610**	4·1622	2·0882	0·0013
415	0·3263	0·0092	1·5611	**615**	3·8863	1·8272	0·0008
420	0·6192	0·0184	2·9743	**620**	3·5349	1·5763	0·0008
425	1·0672	0·0363	5·1628	**625**	3·1074	1·3275	0·0004
430	1·4986	0·0612	7·3139	**630**	2·6548	1·0952	0·0000
435	1·8165	0·0929	8·9747	**635**	2·2358	0·8953	0·0000
440	1·9874	0·1312	9·9687	**640**	1·8468	0·7216	0·0000
445	2·0182	0·1728	10·3351	**645**	1·4909	0·5711	0·0000
450	1·9578	0·2213	10·3194	**650**	1·1743	0·4432	0·0000
455	1·8499	0·2786	10·1235	**655**	0·9058	0·3380	0·0000
460	1·6811	0·3469	9·6497	**660**	0·6807	0·2518	0·0000
465	1·4539	0·4279	8·8481	**665**	0·4965	0·1827	0·0000
470	1·1360	0·5291	7·4860	**670**	0·3542	0·1297	0·0000
475	0·8281	0·6562	6·0719	**675**	0·2548	0·0929	0·0000
480	0·5563	0·8088	4·7305	**680**	0·1846	0·0671	0·0000
485	0·3348	0·9773	3·5571	**685**	0·1270	0·0459	0·0000
490	0·1814	1·1790	2·6369	**690**	0·0855	0·0309	0·0000
495	0·0807	1·4197	1·9396	**695**	0·0581	0·0209	0·0000
500	0·0258	1·7004	1·4319	**700**	0·0408	0·0147	0·0000
505	0·0121	2·0462	1·0665	**705**	0·0283	0·0101	0·0000
510	0·0447	2·4165	0·7600	**710**	0·0197	0·0071	0·0000
515	0·1350	2·8223	0·5183	**715**	0·0136	0·0049	0·0000
520	0·2881	3·2309	0·3559	**720**	0·0093	0·0032	0·0000
525	0·4982	3·6052	0·2604	**725**	0·0062	0·0022	0·0000
530	0·7617	3·9671	0·1942	**730**	0·0042	0·0015	0·0000
535	1·0593	4·2941	0·1399	**735**	0·0029	0·0012	0·0000
540	1·3924	4·5742	0·0973	**740**	0·0020	0·0009	0·0000
545	1·7559	4·7853	0·0654	**745**	0·0014	0·0006	0·0000
550	2·1412	4·9157	0·0430	**750**	0·0008	0·0003	0·0000
555	2·5414	4·9636	0·0283	**755**	0·0005	0·0003	0·0000
560	2·9399	4·9204	0·0193	**760**	0·0005	0·0003	0·0000
565	3·3167	4·7844	0·0132	**765**	0·0003	0·0000	0·0000
570	3·6613	4·5736	0·0101	**770**	0·0003	0·0000	0·0000
575	3·9623	4·3051	0·0085	**775**	0·0000	0·0000	0·0000
580	4·2084	3·9958	0·0078	**780**	0·0000	0·0000	0·0000
				Totals	98·0705	100·0000	118·2246

Table IX. *Leading trichromatic coefficients on the N.P.L., Resolution (1), and C.I.E. systems*

Stimulus	N.P.L.			Resolution (1)			C.I.E.		
	r'	g'	b'	r	g	b	x	y	z
R	1	0	0	1	0	0	0·73467	0·26533	0·00000
G	0	1	0	0	1	0	0·27376	0·71741	0·00883
B	0	0	1	0	0	1	0·16658	0·0088$\underline{6}$	0·82456
W	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0·36463	0·34976	0·28561	0·35046	0·35809	0·2914$\underline{5}$
ϵ	0·30128	0·31409	0·38463	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
S_A	0·52696	0·31940	0·15364	0·5525$\underline{5}$	0·32126	0·12619	0·44757	0·4074$\underline{5}$	0·14498
S_B	0·33054	0·32628	0·34318	0·36230	0·34305	0·2946$\underline{5}$	0·34842	0·35161	0·29997
S_C	0·25190	0·31005	0·4380$\underline{5}$	0·2822$\underline{6}$	0·33326	0·38448	0·3100$\underline{6}$	0·31616	0·37378

Note. When only four decimal places are retained the figures preceding the eight underlined final 5's and 6's are not to be increased.

PART II. APPLICATIONS OF THE STANDARD COLORIMETRIC SYSTEM

Introductory

In the first part of this paper we have endeavoured to present a simple account of the new colorimetric standards and of the way in which they have been derived, together with other information which we believe will be of widespread use. In this part we have to consider the routine operations which are of importance to the industrial worker, and we attach the greatest importance to a clear understanding of the significance of all the operations carried out. The process is partly experimental and partly numerical, and is intimately connected with the procedure adopted in calibrating the instruments used and in deriving the transformation equations for converting the measurements to standard colour coordinates. It is especially important for users to appreciate the implications of this procedure. Reasons of space compel us to confine attention to trichromatic colorimeters.

For convenience we first consider the reasons for utilising, with a trichromatic colorimeter, a working system different from the standard system, and then discuss the transformation equations. To many workers this will appear the most difficult part of the subject. We have therefore thought it advisable to include a fully worked numerical example, following the course which involves the least number of arithmetical processes. By following the steps of this model calculation we hope that the correct procedure will become clear to all readers. In essence we only utilise at each stage relations which are established in dealing with the specially simple conditions encountered in those colorimetric systems which have been of practical importance hitherto. These relations are familiar to many British colorimetrists from previous papers by Ives, Guild, and others. After this account of the transformation equations we resume our consideration of the significance of alternative experimental procedures. For convenience in reference we have separated this discussion into a number of separate topics.

VI. THE WORKING SYSTEM

When a trichromatic colorimeter is calibrated at the National Physical Laboratory the results are not given as numbers for the direct conversion of scale readings to colorimetric coordinates, but as relations connecting the units of the standard system and those of a trichromatic system (constructed in accordance with the principles of Resolution (4)) having as its unitary stimuli the three matching stimuli of the instrument—the working or instrumental system, we may call it. The former term is more widely used, but the latter is useful as it serves to remind the worker that each instrument has a different working system of its own. There are two reasons for giving the calibration in this form. Computation in industrial practice must be reduced to a minimum, and, as more than the minimum labour is involved in converting measurements to a standard basis, only those measurements will usually be converted which are regarded as specially important or are not required only for comparison with other measurements made on the same instrument. Measurements of less importance will normally only be expressed on the instrumental system. The other reason for adopting this form is that, though the colours of the unitary stimuli of the instrument tend to remain unaltered over a long period of time, their relative brightnesses for given instrumental scale readings may be subject to fairly rapid changes arising from causes which need not be discussed here. This inconstancy in the relative significance of the scale readings must be eliminated from the evaluation of the colours before any useful comparison can be made. The course adopted is to re-establish at frequent intervals the relation between the working units and the scale readings by including in any series of measurements readings of the scales for which a colour match is secured with light from a standard source. For both reasons the most convenient general routine is to record measurements in the first place on the instrumental system, and only to convert them to the standard system when necessary. For this conversion equations such as those determined in the course of the N.P.L. calibration of the instrument should be used.

The essential feature of the instrumental system is that the only properties of the instrument and the auxiliary apparatus used with it which affect the system are the colours of the three matching stimuli. The units of the system are determined by assigning definite coordinates to the colour of a standard source. In selecting a source and assigning coordinates it is only necessary to consider the convenience of the user. It will generally be convenient to adopt one of the standard sources *A*, *B*, *C* as the fourth cardinal stimulus, and to make this the basic stimulus of the working system. One advantage of this choice of coordinates is that the units of the working system are found from the most elementary principles and with the least possible calculation. An alternative would be to adopt the plan followed in constructing the C.I.E. standard system, and select those coordinates for the fourth stimulus which happen to make the equi-energy stimulus the basic stimulus. This choice would be less convenient in practice; its only merit is that it does not favour one of the standard stimuli more than the others. Probably each colorimetrist will prefer to adopt as the basic stimulus of his working system that standard source which will be most frequently used in his work. If all three sources are used regularly he will have to decide whether to use a different working system with each illuminant or to use the same system for all three illuminants.

In making a decision the colorimetrist will need to consider the use he intends to make of the results expressed on the instrumental system: for example, if he wishes to be able to compare any colours he has measured on the instrumental system he will probably prefer to employ only one system, though he may be frequently changing the source used for the illumination of specimens. Particular considerations which should influence his experimental routine are discussed later when we come to deal with real observers and the standard observer.

VII. TRANSFORMATION EQUATIONS

(i) *The normal transformation equations*

The measurements made on a colorimeter are only of interest as a means to an end. The end is frequently the specification of a colour on a standard system. The conversion is effected by means of equations of transformation, and it is important to understand how these equations may be derived and how they should be used.

The formation of the equations has been discussed by several writers. Users of the N.P.L. system are probably most familiar with the procedure recommended by Guild (9), which consists in first converting readings on the instrument scales into unit equations on a working system with the illuminant in use (hitherto the N.P.L. white light) as the basic stimulus, secondly finding from the unit equations of the spectral stimuli R, G, B on the working system the number of working system units of these stimuli which match this illuminant, and hence finally forming the equations connecting the working system and the (N.P.L.) reference system. In calibrating an instrument on the new system the procedure corresponds exactly with that hitherto followed, except that the N.P.L. white light is no longer used, one of the new standard illuminants taking its place, and that the quantities of the stimuli R, G, B found in this way are to be inserted in the set of equations given in Section V for the particular illuminant in use. These quantities of R, G, B then disappear from the equations, and the instrumental system is connected directly to the standard system. Thus anyone who has been accustomed to apply the N.P.L. routine will find the changes introduced by the adoption of the new system almost trivial. In the treatment which follows we first give the essential algebraic theory, and subsequently illustrate the working routine by a numerical example.

In accordance with Resolution (4) the approved method of specifying a colorimetric system is to assign definite coordinates to four stimuli (the cardinal stimuli) which can be reproduced accurately in the laboratory. We may wish to convert measurements from one system of this type to another, or we may wish to convert readings taken on the scales of a colorimeter into a standard colorimetric specification. As the equations to be used for both problems are identical in form, we consider in the first place how they are to be obtained.

Let us suppose that U, V, W are three independent stimuli: they may be either the trichromatic units of a known colorimetric system or any convenient units of a particular colorimeter. Let four reproducible stimuli R, G, B, S be matched with mixtures of U, V, W, and the result expressed by the equations

$$\left.\begin{aligned} R &= c_{11}U + c_{12}V + c_{13}W \\ G &= c_{21}U + c_{22}V + c_{23}W \\ B &= c_{31}U + c_{32}V + c_{33}W \\ S &= c_{41}U + c_{42}V + c_{43}W \end{aligned}\right\} \quad \ldots\ldots(7),$$

where the c's are known numbers. The quantities of the four stimuli R, G, B, S denoted by the expressions on the right are not related in a known way. We require to derive expressions for R, G, B in the amounts in which they match the source S; let us denote amounts so related by R_s, G_s, B_s. Now if we eliminate U, V, W between the four equations (7), we obtain an equation which may be written

$$C_4S = C_1R + C_2G + C_3B,$$

and we are clearly at liberty to identify C_1R, C_2G, C_3B with R_s, G_s, B_s respectively.

In this elimination C_4 represents the determinant

$$\begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$$

and C_p ($p = 1, 2, 3$) the determinant formed from this by replacing the pth row by

$$c_{41} \quad c_{42} \quad c_{43}.$$

Multiplying the first three of equations (7) by C_1, C_2, C_3 respectively, we obtain

$$\left.\begin{aligned} R_s = C_1R = C_1c_{11}U + C_1c_{12}V + C_1c_{13}W \equiv \gamma_{11}U + \gamma_{12}V + \gamma_{13}W \\ G_s = C_2G = C_2c_{21}U + C_2c_{22}V + C_2c_{23}W \equiv \gamma_{21}U + \gamma_{22}V + \gamma_{23}W \\ B_s = C_3B = C_3c_{31}U + C_3c_{32}V + C_3c_{33}W \equiv \gamma_{31}U + \gamma_{32}V + \gamma_{33}W \end{aligned}\right\} \ldots(8).$$

Now suppose that on a trichromatic system, of which X, Y, Z are the units, 6
stimuli physically similar to R, G, B, S are given by known numerical equations

$$\left.\begin{aligned} R^* &= d_{11}X + d_{12}Y + d_{13}Z \\ G^* &= d_{21}X + d_{22}Y + d_{23}Z \\ B^* &= d_{31}X + d_{32}Y + d_{33}Z \\ S^* &= d_{41}X + d_{42}Y + d_{43}Z \end{aligned}\right\} \quad \ldots\ldots(9),$$

where the asterisks are introduced to remind us that the quantities are not equal to those given by equations (7). In exactly the same way as before, by eliminating X, Y, Z, we obtain

$$D_4S^* = D_1R^* + D_2G^* + D_3B^*,$$

and we may again identify D_1R^*, D_2G^*, D_3B^* with R_s, G_s, B_s.

Thus we have

$$\left.\begin{aligned} R_s &= D_1 R^* = D_1 d_{11} X + D_1 d_{12} Y + D_1 d_{13} Z \equiv \delta_{11} X + \delta_{12} Y + \delta_{13} Z \\ G_s &= D_2 G^* = D_2 d_{21} X + D_2 d_{22} Y + D_2 d_{23} Z \equiv \delta_{21} X + \delta_{22} Y + \delta_{23} Z \\ B_s &= D_3 B^* = D_3 d_{31} X + D_3 d_{32} Y + D_3 d_{33} Z \equiv \delta_{31} X + \delta_{32} Y + \delta_{33} Z \end{aligned}\right\} \quad (10).$$

By comparing equations (8) and (10) we obtain the transformation equations in the form

$$\left.\begin{aligned} \gamma_{11} U + \gamma_{12} V + \gamma_{13} W &= \delta_{11} X + \delta_{12} Y + \delta_{13} Z \\ \gamma_{21} U + \gamma_{22} V + \gamma_{23} W &= \delta_{21} X + \delta_{22} Y + \delta_{23} Z \\ \gamma_{31} U + \gamma_{32} V + \gamma_{33} W &= \delta_{31} X + \delta_{32} Y + \delta_{33} Z \end{aligned}\right\} \quad \ldots\ldots(11).$$

If these equations are solved for U, V, W, we express the result in the notation

$$\left.\begin{aligned} U &= \omega_{11} X + \omega_{12} Y + \omega_{13} Z \\ V &= \omega_{21} X + \omega_{22} Y + \omega_{23} Z \\ W &= \omega_{31} X + \omega_{32} Y + \omega_{33} Z \end{aligned}\right\} \quad \ldots\ldots(12),$$

and if solved for X, Y, Z by

$$\left.\begin{aligned} X &= \Omega_{11} U + \Omega_{12} V + \Omega_{13} W \\ Y &= \Omega_{21} U + \Omega_{22} V + \Omega_{23} W \\ Z &= \Omega_{31} U + \Omega_{32} V + \Omega_{33} W \end{aligned}\right\} \quad \ldots\ldots(13).$$

Adopting the notation usual in dealing with these mathematical problems, we denote the cofactor of γ_{pq} in the determinant

$$\begin{vmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{vmatrix}$$

by Γ_{qp}, and similarly the cofactor of δ_{pq} in the corresponding δ determinant by Δ_{qp}. By our notation in (13) we have anticipated that Ω_{qp} is the cofactor of ω_{pq}. Then in matrix notation

$$\omega = \Gamma\delta, \quad \Omega = \Delta\gamma,$$

or at length

$$\omega_{pq} = \Gamma_{p1}\delta_{1q} + \Gamma_{p2}\delta_{2q} + \Gamma_{p3}\delta_{3q} \quad (p,\ q = 1,\ 2,\ 3) \qquad \ldots\ldots(14),$$

$$\Omega_{pq} = \Delta_{p1}\gamma_{1q} + \Delta_{p2}\gamma_{2q} + \Delta_{p3}\gamma_{3q} \quad (p,\ q = 1,\ 2,\ 3) \qquad \ldots\ldots(15).$$

We note that it is not necessary to evaluate any of the third order determinants denoted by Greek letters, for we are only concerned with the mutual ratios of any set of coefficients, not with their absolute values.

When an instrument is calibrated by the indirect method the formation of the equations connecting the standard and the working systems is simpler. Indirect colorimetry† gives unit equations for some unknown quantities U', V', W' of the instrumental stimuli in the form

$$\begin{aligned} U' &= a_{11} X + a_{12} Y + a_{13} Z \\ V' &= a_{21} X + a_{22} Y + a_{23} Z \\ W' &= a_{31} X + a_{32} Y + a_{33} Z, \end{aligned}$$

---

† For explanation of indirect colorimetry see Section IV.

and to these we add the known coordinates of the source selected as the basic stimulus of the working system

$$S = a_{41}X + a_{42}Y + a_{43}Z.$$

As before, the elimination of X, Y, Z gives

$$A_4 S = A_1 U' + A_2 V' + A_3 W',$$

and we identify A_1U', A_2V', A_3W' with U, V, W, the units of the instrumental system. Multiplying the first three equations throughout by A_1, A_2, A_3, we obtain the direct equations

$$\begin{aligned} U &= \omega_{11}X + \omega_{12}Y + \omega_{13}Z \\ V &= \omega_{21}X + \omega_{22}Y + \omega_{23}Z \\ W &= \omega_{31}X + \omega_{32}Y + \omega_{33}Z, \end{aligned}$$

and the reverse transformation is obtained by computing cofactors.

On the N.P.L. system the ultimate variables X, Y, Z are identical with R_s, G_s, B_s, so that in equations (11) $\delta_{11} = \delta_{22} = \delta_{33} = 1$, and all the other δ's vanish. One set of equations, viz. that giving X, Y, Z in terms of U, V, W, is therefore already found, and the reverse set follows on simply evaluating the Γ's. In the more general case there are three ways of arranging the numerical work in deriving the transformation equations:

(i) computing the Γ's from the γ's and using equations (14) and (15) to obtain the ω's and Ω's;

(ii) computing the Γ's from the γ's, using equation (14) to obtain the ω's and computing the Ω's from the ω's;

(iii) using equations (15) to find the Ω's and computing the ω's from the Ω's.

When calculations of this kind are frequently called for, it is important to adopt the plan which involves least labour. As the *d*'s of equations (9) are fixed numbers, the δ's and Δ's may be calculated once for all. They have already been given for the new standard system and the three standard illuminants in Section V. In consequence these three mathematically equivalent procedures involve calculations of different length. The first is the longest, and the third the shortest. It is evident that the calculation of the Γ's on the second plan represents work which is wholly additional to the processes involved in the third. The second is superior to the first because multiplying two matrices is longer than finding a reciprocal. In multiplication there are 27 multiplications and 18 additions; in finding the reciprocal there are 18 multiplications and 9 additions. We shall suppose in the numerical example we give that plan (iii) is to be followed.

In calculating C_1, C_2, C_3 the longest way is to find C_{qp}, the cofactor of c_{pq} in the determinant C_4, using the relation

$$C_p = c_{41}C_{1p} + c_{42}C_{2p} + c_{43}C_{3p}.$$

A better way, if equations (7) are unit equations, is to take

$$\left.\begin{aligned} C_1 &= (c_{2r} - c_{4r})(c_{3s} - c_{4s}) - (c_{3r} - c_{4r})(c_{2s} - c_{4s}) \\ C_2 &= (c_{3r} - c_{4r})(c_{1s} - c_{4s}) - (c_{1r} - c_{4r})(c_{3s} - c_{4s}) \\ C_3 &= (c_{1r} - c_{4r})(c_{2s} - c_{4s}) - (c_{2r} - c_{4r})(c_{1s} - c_{4s}) \end{aligned}\right\} \qquad \ldots\ldots(16),$$

where r and s are two definite integers selected from 1, 2, 3. If equations (7) represent scale readings, the shortest course is to divide each equation throughout by the coefficient of one of its variables—for example the coefficient of V. In the modified equations the coefficient of V is then unity, and C_1, C_2, C_3 are calculated by the formulae just given where $r = 1$, $s = 3$.

Numerical example

Suppose the matches made on the cardinal stimuli of the standard system are given by

$$\begin{aligned} R &= 84{\cdot}59U_i - 5{\cdot}09V_i + 0{\cdot}00W_i \\ G &= 7{\cdot}29U_i + 86{\cdot}97V_i - 7{\cdot}32W_i \\ B &= 2{\cdot}42U_i - 2{\cdot}64V_i + 78{\cdot}70W_i \\ S_B &= 41{\cdot}81U_i + 40{\cdot}80V_i + 80{\cdot}15W_i, \end{aligned}$$

where the suffix i to U, V, W indicates that the units are those depending on the immediate instrumental indicators. The first step will usually be to change the units to U, V, W where equal quantities of U, V, W match S_B. We therefore divide the figures of the three columns by the coefficients of the last equation, obtaining

$$\begin{aligned} R &= 2{\cdot}0232U - 0{\cdot}1248V + 0{\cdot}0000W \\ G &= 0{\cdot}1744U + 2{\cdot}1316V - 0{\cdot}0913W \\ B &= 0{\cdot}0579U - 0{\cdot}0647V + 0{\cdot}9819W \\ S_B &= 1{\cdot}0000U + 1{\cdot}0000V + 1{\cdot}0000W, \end{aligned}$$

or, on unification,

$$\begin{aligned} R &= 1{\cdot}0657U - 0{\cdot}0657V + 0{\cdot}0000W \\ G &= 0{\cdot}0787U + 0{\cdot}9625V - 0{\cdot}0412W \\ B &= 0{\cdot}0594U - 0{\cdot}0664V + 1{\cdot}0070W \\ S_B &= \tfrac{1}{3}\,U + \tfrac{1}{3}\,V + \tfrac{1}{3}\,W. \end{aligned}$$

So far we have only illustrated a common procedure for finding coordinates on the instrumental system from the initial scale readings. The figures of these unit equations obviously represent the original observations with adequate accuracy. In the succeeding stages it is advisable to include an additional figure to minimise the effect of a succession of operations in giving erroneous values to the last figure. This additional figure will be dropped when the calculations are completed. Adding then 0 as a fifth decimal place in the equations for R, G, B, and taking 0·33333 as the coordinates for S_B, we have, if we select the second and third coordinates for the calculation of C_1, C_2, C_3 by equations (16), for the coefficients diminished by those of S_B the numbers

$$\begin{array}{rr} -\,0{\cdot}39903 & -\,0{\cdot}33333 \\ +\,0{\cdot}62917 & -\,0{\cdot}37453 \\ -\,0{\cdot}39973 & +\,0{\cdot}67367 \end{array}$$

and therefore

$$\begin{aligned} C_1 &= 0{\cdot}62917 \times 0{\cdot}67367 - (-\,0{\cdot}39973)(-\,0{\cdot}37453) = 0{\cdot}27414 \\ C_2 &= (-\,0{\cdot}39973) \times (-\,0{\cdot}33333) - (-\,0{\cdot}39903) \times (0{\cdot}67367) = 0{\cdot}40206 \\ C_3 &= (-\,0{\cdot}39903) \times (-\,0{\cdot}37453) - 0{\cdot}62917 \times (-\,0{\cdot}33333) = 0{\cdot}35917. \end{aligned}$$

If we multiply our equations for R, G, B by these factors we shall obtain smaller coefficients than are desirable. We therefore divide C_1 and C_3 by C_2 and obtain the equivalent multipliers 0·68184, 1·00000, 0·89333. Carrying out the multiplications, and replacing R, G, B by R_B, G_B, B_B as an indication that a combination of one of each of the new units matches standard illuminant B, we obtain

$$\begin{array}{rl}
R_B = & 0{\cdot}72664U - 0{\cdot}04480V + 0{\cdot}00000W \\
G_B = & 0{\cdot}07870U + 0{\cdot}96250V - 0{\cdot}04120W \\
B_B = & 0{\cdot}05306U - 0{\cdot}05932V + 0{\cdot}89958W \\
\hline
 & 0{\cdot}85840 \quad\; 0{\cdot}85838 \quad\; 0{\cdot}85838
\end{array}$$

At this stage we can check the accuracy of the calculations by seeing that the sums of the coefficients of U, V, W respectively only differ by trifling amounts*. These values of R_B, G_B, B_B are to be substituted in the equations giving X, Y, Z in terms of these variables in Section V. The calculations are as follows:

$$\begin{array}{rrrr}
 & 1{\cdot}62690 & -\,0{\cdot}10030 & 0{\cdot}00000 \\
 & -\,0{\cdot}04054 & -\,0{\cdot}49583 & +\,0{\cdot}02122 \\
 & +\,0{\cdot}00032 & -\,0{\cdot}00036 & +\,0{\cdot}00545 \\
\hline
X = & 1{\cdot}58668U & -\,0{\cdot}59649V & +\,0{\cdot}02667W \\
 & -\,0{\cdot}61684 & +\,0{\cdot}03803 & 0{\cdot}00000 \\
 & +\,0{\cdot}11226 & +\,1{\cdot}37291 & -\,0{\cdot}05877 \\
 & -\,0{\cdot}00089 & +\,0{\cdot}00099 & -\,0{\cdot}01509 \\
\hline
Y = & -\,0{\cdot}50547U & +\,1{\cdot}41193V & -\,0{\cdot}07386W \\
 & -\,0{\cdot}32204 & +\,0{\cdot}01985 & 0{\cdot}00000 \\
 & +\,0{\cdot}00698 & +\,0{\cdot}08542 & -\,0{\cdot}00366 \\
 & +\,0{\cdot}06234 & -\,0{\cdot}06970 & +\,1{\cdot}05699 \\
\hline
Z = & -\,0{\cdot}25272U & +\,0{\cdot}03557V & +\,1{\cdot}05333W
\end{array}$$

These calculations also can be immediately checked by multiplying the equations respectively by 0·34842, 0·35161, 0·29997 (the coefficients of S_B on the standard system) and adding. The sum gives

$$0{\cdot}34842X + 0{\cdot}35161Y + 0{\cdot}29997Z = 0{\cdot}29929U + 0{\cdot}29929V + 0{\cdot}29929W$$

and the agreement in the coefficients on the right confirms the accuracy of the arithmetic. From these equations for X, Y, Z we obtain the reverse transformation by finding the cofactors.

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* The coefficients in equations of this type are frequently made definite by arbitrarily choosing unity as the value of the sums. This means that the S_B which matches $\frac{1}{3}R_B+\frac{1}{3}G_B+\frac{1}{3}B_B$ represents the same amount of energy as the S_B which matches $\frac{1}{3}U+\frac{1}{3}V+\frac{1}{3}W$. This convention is quite arbitrary, and it will often be simpler to dispense with it.

Thus

$$
\begin{aligned}
\omega_{11} &= 1{\cdot}41193 \times 1{\cdot}05333 - 0{\cdot}03557 \times (-\,0{\cdot}07386) = 1{\cdot}48986\\
\omega_{12} &= -\,0{\cdot}03557 \times 0{\cdot}02667 - (-\,0{\cdot}59649) \times 1{\cdot}05333 = 0{\cdot}6292\underline{5}\\
\omega_{13} &= -\,0{\cdot}59649 \times (-\,0{\cdot}07386) - 1{\cdot}41193 \times 0{\cdot}02667 = 0{\cdot}00640\\
\omega_{21} &= -\,0{\cdot}25272 \times (-\,0{\cdot}07386) - (-\,0{\cdot}50547) \times 1{\cdot}05333 = 0{\cdot}55109\\
\omega_{22} &= 1{\cdot}58668 \times 1{\cdot}05333 - (-\,0{\cdot}25272) \times 0{\cdot}02667 = 1{\cdot}67804\\
\omega_{23} &= -\,(-\,0{\cdot}50547) \times 0{\cdot}02667 - 1{\cdot}58668 \times (-\,0{\cdot}07386) = 0{\cdot}10371\\
\omega_{31} &= -\,0{\cdot}50547 \times 0{\cdot}03557 - (-\,0{\cdot}25272) \times 1{\cdot}41193 = 0{\cdot}33884\\
\omega_{32} &= -\,0{\cdot}25272 \times (-\,0{\cdot}59649) - 1{\cdot}58668 \times 0{\cdot}03557 = 0{\cdot}09431\\
\omega_{33} &= 1{\cdot}58668 \times 1{\cdot}41193 - (-\,0{\cdot}50547) \times (-\,0{\cdot}59649) = 1{\cdot}93877.
\end{aligned}
$$

To check these figures we sum the coefficients of X, Y, Z separately. Thus

$$
\begin{array}{ccc}
1{\cdot}48986 & 0{\cdot}62925 & 0{\cdot}00640\\
0{\cdot}55109 & 1{\cdot}67804 & 0{\cdot}10371\\
0{\cdot}33884 & 0{\cdot}09431 & 1{\cdot}93877\\
\hline
2{\cdot}37979 & 2{\cdot}40160 & 2{\cdot}04888
\end{array}
$$

Dividing the others by the sum of the coefficients of Y, we find

$$\frac{2{\cdot}37979}{2{\cdot}40160} = 0{\cdot}99092, \quad \frac{2{\cdot}04888}{2{\cdot}40160} = 0{\cdot}85313$$

and the agreement with the totals of Table VII confirms the accuracy of the work. The results we have reached will be given by the alternative systems of equations

$$
\begin{aligned}
U &= 1{\cdot}4899\,X + 0{\cdot}6292\,Y + 0{\cdot}0064\,Z\\
V &= 0{\cdot}5511\,X + 1{\cdot}6780\,Y + 0{\cdot}1037\,Z\\
W &= 0{\cdot}3388\,X + 0{\cdot}0943\,Y + 1{\cdot}9388\,Z
\end{aligned}
$$

and

$$
\begin{aligned}
X &= \,1{\cdot}5867U - 0{\cdot}5965V + 0{\cdot}0267W\\
Y &= -\,0{\cdot}5055U + 1{\cdot}4119V - 0{\cdot}0739W\\
Z &= -\,0{\cdot}2527U + 0{\cdot}0356V + 1{\cdot}0533W.
\end{aligned}
$$

It will be observed that the one set of equations is the equivalent of the other, but as we have not evaluated the determinants the one is not the formal solution of this other. In fact if we multiply the two matrices together we obtain 2·04424 as the value of the lower determinant. There is no advantage to be gained by multiplying each term of the former by $(2{\cdot}04424)^{-\frac{2}{3}}$ and of the latter by $(2{\cdot}04424)^{-\frac{1}{3}}$. By inspection of the magnitudes in the leading diagonals we can see that each set of equations is suitable for use, and no additional labour spent on them is profitable.

As it is usually desired in the calibration of instruments to make it clear that the units U, V, W are red, green, blue respectively, we shall replace these letters by R, G, B. For the rest of this section these letters without a suffix will denote the units of the instrumental unitary stimuli. To denote units which merely depend upon the instrumental scale readings we shall write R_i, G_i, B_i. The corresponding coordinates on the instrumental system will be denoted by r, g, b, so that the unit equation of a stimulus Q on the instrumental system will be

$$Q = rR + gG + bB.$$

When the units are given by the scale readings the corresponding equation will be

$$Q = \rho R_i + \gamma G_i + \beta B_i.$$

The units of the international standard system defined in Resolution (5) will be denoted by black letter type $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{Z}$. The unit equation of a colour on this system will usually be written in the form

$$\mathbf{Q} = x\mathbf{X} + y\mathbf{Y} + z\mathbf{Z}.$$

(ii) *Derivation of scale-reading transformations*

The usual practice, as has already been stated, is to convert measurements immediately from scale readings to coordinates on a definite instrumental system. It may sometimes happen that it is known, before any arithmetical work is started, that several measurements on different colours made on a single occasion will be required on the standard system. Under these conditions labour is reduced (provided there are more than three colours) by converting the transformation equations into a form in which scale readings may be inserted directly. Let the usual calibration equations be

$$\begin{aligned} R &= \omega_{11}\mathbf{X} + \omega_{12}\mathbf{Y} + \omega_{13}\mathbf{Z} \\ G &= \omega_{21}\mathbf{X} + \omega_{22}\mathbf{Y} + \omega_{23}\mathbf{Z} \\ B &= \omega_{31}\mathbf{X} + \omega_{32}\mathbf{Y} + \omega_{33}\mathbf{Z} \end{aligned}$$

and suppose that on the instrumental system r, g, b and r_0, g_0, b_0 are the coordinates of any colour under measurement and of the standard source used for the current determination of the units respectively. Let the corresponding scale readings be ρ, γ, β and ρ_0, γ_0, β_0 respectively. Then we clearly have

$$r = \rho\frac{r_0}{\rho_0}, \quad g = \gamma\frac{g_0}{\gamma_0}, \quad b = \beta\frac{b_0}{\beta_0},$$

so that since

$$rR + gG + bB = \rho R_i + \gamma G_i + \beta B_i,$$

the equations required are

$$\begin{aligned} R_i &= \omega'_{11}\mathbf{X} + \omega'_{12}\mathbf{Y} + \omega'_{13}\mathbf{Z} \\ G_i &= \omega'_{21}\mathbf{X} + \omega'_{22}\mathbf{Y} + \omega'_{23}\mathbf{Z} \\ B_i &= \omega'_{31}\mathbf{X} + \omega'_{32}\mathbf{Y} + \omega'_{33}\mathbf{Z}, \end{aligned}$$

where

$$\omega'_{1p} = \frac{r_0}{\rho_0}\omega_{1p}, \quad \omega'_{2p} = \frac{g_0}{\gamma_0}\omega_{2p}, \quad \omega'_{3p} = \frac{b_0}{\beta_0}\omega_{3p} \quad (p = 1, 2, 3).$$

(iii) *The conversion of instrumental readings to coordinates on the instrumental system*

If the instrumental system has the source in use represented by the centroid of the instrumental colour triangle the conversion of readings to instrumental system coordinates can be effected immediately. If ρ_0, γ_0, β_0 are scale readings when the standard source is matched, the coordinates corresponding to the scale readings ρ, γ, β are

$$c\frac{\rho}{\rho_0}, \quad c\frac{\gamma}{\gamma_0}, \quad c\frac{\beta}{\beta_0},$$

where $c^{-1} = \frac{\rho}{\rho_0} + \frac{\gamma}{\gamma_0} + \frac{\beta}{\beta_0}$. In practice the readings $\rho_0, \gamma_0, \beta_0$ are usually multiplied by a factor—say $1/\rho_0$—to eliminate one multiplication for each colour when several measurements are to be made.

When the source used for the daily calibration of the instrument is not represented by the centroid of the instrumental triangle, it becomes necessary to make use of the calibration equations of the instrument to determine the factors for expressing the results on the instrumental system. Taking these equations in the standard form

$$\begin{aligned} R &= \omega_{11}\mathbf{X} + \omega_{12}\mathbf{Y} + \omega_{13}\mathbf{Z}, & \mathbf{X} &= \Omega_{11}R + \Omega_{12}G + \Omega_{13}B \\ G &= \omega_{21}\mathbf{X} + \omega_{22}\mathbf{Y} + \omega_{23}\mathbf{Z}, & \mathbf{Y} &= \Omega_{21}R + \Omega_{22}G + \Omega_{23}B \\ B &= \omega_{31}\mathbf{X} + \omega_{32}\mathbf{Y} + \omega_{33}\mathbf{Z}, & \mathbf{Z} &= \Omega_{31}R + \Omega_{32}G + \Omega_{33}B \end{aligned}$$

suppose x_0, y_0, z_0 are the coordinates on the standard system of the calibrating source used. From the second set of equations its coordinates on the instrumental system are proportional to $\Omega_1, \Omega_2, \Omega_3$, where

$$\Omega_p = x_0\Omega_{1p} + y_0\Omega_{2p} + z_0\Omega_{3p} \quad (p = 1, 2, 3).$$

If the scale readings on this source are $\rho_0, \gamma_0, \beta_0$, and those for another colour are ρ, γ, β, the coordinates of this colour on the instrumental system are proportional to $\rho\Omega_1/\rho_0, \gamma\Omega_2/\gamma_0, \beta\Omega_3/\beta_0$ and the actual values are determined from these by unification. As in the preceding section, we may alter all these factors in any convenient common ratio and thus reduce the number of computational processes by one.

(iv) *Change of basic stimulus in the working system*

The transformation equations may have been given in a form which assumes that one particular stimulus is represented at the centroid of the instrumental triangle, and it may be desired to derive equations for a different basic stimulus on the working system. Let x_c, y_c, z_c be the coordinates on the standard system of the new working basic stimulus. As in the previous section, the coordinates of this source on the existing working system are proportional to $\Omega_1, \Omega_2, \Omega_3$, where

$$\Omega_p = x_c\Omega_{1p} + y_c\Omega_{2p} + z_c\Omega_{3p}.$$

It immediately follows that if the coefficients of the new transformation equations are distinguished by accents

$$\omega'_{pq} = \Omega_p\omega_{pq}, \quad \Omega'_{pq} = \Omega_{pq}/\Omega_q \qquad \text{......(17), (18).}$$

Rather more generally we may alter the transformation to secure coordinates

r_c, g_c, b_c on the instrumental system to correspond to x_c, y_c, z_c on the standard system. We then define Ω_1, Ω_2, Ω_3 by

$$\begin{aligned} r_c\Omega_1 &= x_c\Omega_{11} + y_c\Omega_{21} + z_c\Omega_{31} \\ g_c\Omega_2 &= x_c\Omega_{12} + y_c\Omega_{22} + z_c\Omega_{32} \\ b_c\Omega_3 &= x_c\Omega_{13} + y_c\Omega_{23} + z_c\Omega_{33} \end{aligned}$$

and ω'_{pq}, Ω'_{pq} are computed as in (17) and (18).

VIII. REAL OBSERVERS AND THE STANDARD OBSERVER

The standard observer defined in Resolution (1) has been adopted to enable agreement to be reached in the specification of colour, without restricting the methods employed in reaching this specification or the persons by whom it is derived. The properties required to define this (or any other) observer are his visibility function, his colour coordinates for the complete spectrum, and his factors for the luminosity of the trichromatic units. Direct colorimetry is necessarily practised by observers whose colour vision differs from that of the standard observer. Not only is our knowledge of a particular observer in general incomplete, but it is subject to temporary variations. Since all standard colorimetry aims at reaching the evaluation of colours as they are seen by the standard observer, we are much interested in the differences between colour matches made by real observers and by the standard observer. For convenience we call the difference between the determinations of the colour of a given stimulus by a real observer and by the standard observer the systematic personal error of the real observer. In general our knowledge of these personal errors is far less complete than we should like it to be; this is because our knowledge of real observers is very defective.

The principal effects of systematic personal errors can be realised if we imagine the real and the standard observer making settings on groups of stimuli. If the stimuli of each group appear of the same colour to the standard observer, and include representatives of the whole range of physical structure compatible with this condition, a real observer whose colour vision differs from that of the standard observer will make rather scattered instrumental settings on the group as a whole and the range will include the settings of the standard observer. Thus, in general, we cannot relate the personal error of the real observer to the colour of the group.

In the very important branch of colorimetry which is concerned with the measurement of the colours of materials, the scattering is restricted because we are chiefly concerned with a sub-group. A relation then exists between personal error and colour, which, though not exact or universally applicable, nevertheless affords a very useful test by which to weigh the relative merits of different experimental procedures. It is exceptional for the radiation transmitted or reflected by commercial materials to be confined to a number of narrow spectral regions. Stimuli of a given colour tend towards a particular spectral distribution of energy which is, roughly, the most uniform diffusion of energy in the spectrum compatible with the colour in question. For this reason the majority of commercial materials which would be grouped together by the standard observer as having the same colour would also be grouped together by the real observer. It does not follow that the ratios of the scale readings obtained by the real and standard observers would agree. The colour is matched by a mixture of the three instrumental stimuli, and, if the colorimeter is intended for general work, these stimuli must be somewhat highly saturated. A mixture of the three is therefore characterised by three more or less pronounced spectral peaks, and this stimulus tends to differ markedly in spectral structure from the type of distribution which most frequently corresponds to the particular colour, especially when this is of low saturation. Under these conditions the differences in the visual properties of the real and standard observers are likely to result in a personal error of appreciable magnitude, but this error will be nearly constant for all members of the sub-group on account of their spectral similarity. When we are only dealing with commercial materials, we shall therefore almost always find that personal errors are closely determined by the colours which are being measured.

Let us suppose that the unitary stimuli of the colorimeter are obtained by means of colour filters and a source yielding a continuous spectrum. These filters, since they owe their colour to typical dyes or colouring matter, give to the individual unitary stimuli spectral characteristics belonging to the same group as the majority of coloured materials of the same or similar colours. It will therefore generally happen that any specimen colour which is close to one or other of the instrumental stimuli is physically similar to it, and will therefore be matched in the same way by all observers. We may express this by saying that the personal error of any real observer tends to be zero for colours which match any one of the unitary stimuli of the instrumental system.

On the other hand, the white and pale colours of commerce are mostly either non-selective, or only slightly selective, and are thus physically strongly differentiated from the peaked matching stimulus. The personal errors, judged by instrumental settings, are therefore likely to be greatest for white and pale colours*.

Now white and pale materials are among the most important commercial products, and it is desirable to adopt a technique which reduces personal errors to a minimum for these colours. This can be done by each observer determining for himself the scale factors for reducing his readings to the instrumental system from a match of the standard source which is to be used to illuminate specimens†. Since the spectral structure of the standard illumination is identical with that of the light reflected from a non-selective specimen, to which all the "whites" of commerce approximate, this procedure exactly neutralises the personal error for these materials, and greatly reduces it for others.

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* It may be mentioned in passing that the personal errors present in the colorimetry of materials will in general be less when the matching stimuli are produced by filters than when they are homogeneous spectral radiations. The spectral structure of a matching stimulus composed of three homogeneous stimuli necessarily differs far more from that reflected or transmitted by any material than does that composed of the broader spectral bands transmitted by filters. The distribution of errors will be determined by similar considerations in both cases, but the greater contrast between the stimuli matched in the spectral instrument will result in larger personal errors.

† In practice it is convenient, but not necessary, in matching a standard light source to reflect the light from a surface of magnesium oxide which is known to be practically non-selective.

In terms of the distribution of errors over the colour field we may say that measurements based simply on the scale readings result in personal errors which are zero or very small at the corners of the working triangle, but increase to a maximum in the white region of the triangle. The same result follows if the scale factors for reducing the scale readings are not those found by the particular observer who made the matches. On the other hand, if the measurements are reduced to units based on the observer's own match of the illuminant, the error is corrected at four points, namely at the corners and the point representing the illuminant, and as the latter is near the centre of the triangle these four points are very favourably situated for reducing the errors in the field as a whole.

As far as the personal errors of observation are concerned it would make little difference in practice whether the scale factors of the instrument were determined from the particular standard source to be used for illuminating the specimens under test, or from another, and on purely metrological grounds there would be a theoretical advantage in always employing standard illuminant *B* for determining these factors, since it is one of the cardinal stimuli of the standard reference system. This would give a distribution of errors of a constant character with respect to position in the triangle whichever source was being used to measure colours. There are practical considerations which outweigh this. The illuminants have to be set up from specifications involving the physical properties of lamps and liquid filters. There may be systematic errors in the procedure adopted in any laboratory for setting up these standards, and in addition there may be accidental day to day variations in the standards. A departure in the properties of the illuminant as used, from those which it is supposed to have, produces errors in the measurements. These errors will be least for highly selective materials reflecting or transmitting a single narrow spectral band, and will be greatest for white and pale materials. If the scale factors of the instrument are determined for the illuminant as actually set up on the particular occasion, the errors due to any departure from its assumed properties are completely compensated for white and pale specimens and considerably reduced for others. Thus, both on the ground of reducing the personal errors of observation and of reducing errors arising from defective standardisation of the light sources, it is desirable for each observer as a regular part of his daily routine to determine the scale factors. This ensures that the actual standard illuminant he is using occupies its theoretically correct position in the instrumental colour triangle. The only occasions on which it seems desirable to deviate from this rule are when an unscreened lamp, such as standard illuminant *A*, or some other special low temperature source is used to illuminate the specimens. Owing to the yellowness of these sources the accuracy with which the blue scale factor can be determined by this procedure is much less than when one of the filtered sources *B* or *C* is used. Errors in this factor may give rise to errors in the measurement of blue specimens exceeding those likely to arise from inaccuracy in the properties of the sources. It is suggested, therefore, as a safe working rule, that when specimens are examined in the light from standard illuminant A, the scale factors should be determined from observations on A, provided the specimens are either pale or of dominant hue within the long wave-length half of the spectrum, but from observations on standard illuminant B when the specimens are predominantly blue.

As mentioned in Section VI, a separate working system may be employed for measurements with each illuminant; each illuminant may then be the basic stimulus of its own system. If this procedure is adopted, a separate set of transformation equations must be employed to relate each of these working systems to the standard reference system. As a rule it will probably be found more convenient to adopt a single working system for use with all illuminants. Only one set of transformation equations is then necessary. The use of a single working system does not affect the general practice of determining the scale factors by matching whichever illuminant is in use. In order to determine these factors from a match of a source which is not the basic stimulus of the working system we must find the coordinates of this source on the working system. These are readily obtained from the equations relating the working system to the standard reference system. The required factors are numbers which reduce the scale readings obtained when matching the source to proportionality to these coordinates (see Section VII (iii)).

To summarise the foregoing discussion, it has been shown that, for several distinct reasons, certain of the calibration constants of a colorimeter should be determined as a regular part of colorimetric routine, and that each observer should determine them for himself. In practice this daily calibration involves little trouble, consisting of nothing more than the measurement of one extra colour, namely that of the illuminant in use, and in return for this trifling additional labour we eliminate errors due to accidental or progressive changes in the impermanent properties of the instrument (see Section VI) and secure a considerable degree of compensation of personal errors of the observer, and of errors arising from faulty setting up of the standard illuminants.

The question then arises whether it is desirable, from the point of view of reducing the errors in results expressed on the standard reference system, for each observer to determine the transformation constants also for himself, or whether, having obtained the results on the instrumental system by means of his own scale factors, he should transform these results to the standard reference system by means of transformation equations characteristic of the standard observer.

The difference in the results obtained by adopting one or other of these alternatives is easily seen. By the procedure we have described any real observer obtains on the working system of the instrument results identical with those of the standard observer for the four colours represented by the unitary stimuli of the instrument and the standard source from which he has determined his scale factors. If he transforms these results by the standard observer's equations these four colours will also be correct on the standard system. If on the other hand the real observer uses transformation equations derived from his own observations of the cardinal stimuli of the standard system, he automatically obtains the correct standard coordinates for the colours of the three spectral cardinal stimuli and of the standard illuminant used to determine the scale factors.

The difference in the distribution of errors is thus indicated by zero values for the colours of the instrumental stimuli when the standard observer's equations are used, and for the three spectral colours when the real observer's equations are used. In any practical case the difference will be slight, for the unitary stimuli of colorimeters designed for general work are never very far removed from the spectral stimuli of the standard system. The relative positions of the cardinal and the instrumental stimuli with respect to the part of the colour field of predominant commercial importance are the most significant factors in any comparison. As the cardinal stimulus *G* is distinctly yellower than any of the green stimuli which give all the coordinates positive over the greatest portion of the commercially important field, a slightly better distribution of errors will generally be obtained if the compensation is effected for the colours of the instrumental stimuli rather than for the cardinal stimuli. The real observer should accordingly use the transformation equations of the standard observer, rather than his own. Practical convenience also points in the same direction, for, unlike the determination of scale factors, the measurements on which the permanent calibration rests require greater skill and experience than is required in the routine use of a colorimeter. The facts that the equations involve only the relatively permanent properties of the instrument, and that they need not be determined by each user, make it possible for the calibration to be carried out at a standardising institution, thus relieving the staff of industrial laboratories, where suitable auxiliary equipment may not be available, from the necessity of carrying out such work.

When calibrations are carried out at the National Physical Laboratory the results will normally be given as transformation equations relating the units of the C.I.E. standard system to those of an instrumental system having standard illuminant *B* as the basic stimulus, but any user who wishes to employ an instrumental system with a different basic stimulus can easily make the necessary alterations to the instrumental units as explained in the section dealing with the derivation of the transformation equations.

IX. CALIBRATION OF TRICHROMATIC COLORIMETERS

It has already been pointed out under the heading "the working system" that, owing to the dependence of the scale readings of a colorimeter on the adjustment of the lamp or other parts of the instrument and on other factors, a calibration which transforms scale readings directly to standard coordinates is only valid at the time it was made. On the other hand, as the colours of the instrumental stimuli may be taken as invariable over comparatively long periods, a calibration which connects an instrumental trichromatic system with the standard system is comparatively stable. The calibration in practice has therefore to be separated into two parts. The variable factors are determined by the observer at frequent intervals, preferably every time the instrument is used, and by means of this calibration the scale readings are converted to coordinates on the relatively stable instrumental system. We are only concerned in this section with the procedure to be followed in carrying out the more permanent part of the calibration, which will only require to be checked at long intervals.

Of the two general methods available for effecting this calibration, the direct method, based on colour matches of the four cardinal stimuli of the standard system, leads to equations valid for the observers who made the matches, and the indirect method, based on the spectrophotometry of the filters, a measurement of the colour temperature of the lamp fitted in the instrument, and the properties of the standard observer, gives equations valid for the standard observer. We give reasons elsewhere for preferring the equations obtained by the standard observer; to approach as closely as possible to these equations when the direct method is used measurements should be made by several observers known to be approximately normal.

In applying the direct method a suitable mercury lamp should be used to give the green and blue cardinal stimuli. The Wratten series of filters includes some specially designed for isolating these spectral lines. Particularly with the blue stimulus the use of two filters in series is recommended. The numbers recommended are, for the green stimulus 77*A* and 65 and for the blue 50 and 35. The red stimulus is defined as having a wave-length of 700 mμ, but as the hue is constant for wave-lengths greater than about 670 mμ, a convenient procedure is to use a gas-filled lamp in combination with a very deep red filter, such as Ilford "spectrum red." When colour matches on all four cardinal stimuli have been made by a suitable number of observers, the calculations are carried out as described.

The transformation can readily be computed for any desired instrumental basic stimulus for which the coordinates on the standard system are known. The usual practice at the National Physical Laboratory will probably be to choose standard illuminant *B*, one of the cardinal stimuli of the standard system, as the basic stimulus of the working system. This choice preserves effective continuity with past practice in Great Britain.

When the colours of the working unitary stimuli are computed from the properties of the standard observer, the spectrophotometry must be carried out with care, and the spectrophotometer must be properly calibrated. At those parts of the spectrum where the transmission of a filter changes rapidly the need for slit-width corrections must be considered.

The accuracy necessary in the determination of the colour temperature of the colorimeter lamp varies with the nature of the filters, and the latitude permissible in use depends on the same factors. When the filters are highly selective no great exactitude in the lamp temperature is necessary, and it will frequently be sufficient to assume the colour temperature given by the lamp maker as the typical value for lamps of the type selected. On the other hand, when filters transmitting over broad spectral regions are used it is necessary to compute their colour coordinates for a black-body source operating at a definite colour temperature, and in the use of the instrument to control the lamp carefully so that it operates at this particular colour temperature.

When the calibration is carried out in this way it must not be forgotten that the light reaching the observer's eye may be modified by spectral variation in the transmission of optical parts other than the filters. The instrument must be examined to make sure that suitable glass has been used for these parts.

X. LUMINOSITY FACTORS OF INSTRUMENTAL UNITS

Among the measurements frequently made with the colorimeter are determinations of the brightness factors of materials. They involve a knowledge of the luminosity factors L_R, L_G, L_B of the unit stimuli on the working system.

Values of these factors, or rather their ratios, the only magnitudes of significance, can be derived formally from the transformation equations. Since the unitary stimuli corresponding to X and Z on the standard system have zero luminosity for the standard observer, the luminosity factors of the working units should be given by the coefficients of Y in the transformation equations for these units. This procedure is open to objection, as has been previously shown by Guild (4). Whatever method of calibration may be used, the transformation equations for converting measurements on the working system to the standard system are affected by experimental error. The calibration is satisfactory if any errors in colour coordinates resulting from errors in calibration are colorimetrically insignificant. It is quite possible for this condition to be satisfied and yet for the calibration errors to give relative luminosity factors which are not invariably as accurate as is required. For this reason it has been the practice in calibrating instruments on the N.P.L. system to determine luminosity factors directly from measurements on filters which have had their transmission factors measured by a direct method. Before values based on the transformation equations are adopted it is always desirable to make measurements of this type to check their reliability.

XI. INDIRECT COLORIMETRY

It has already been pointed out that with the adoption of a standard observer it has become possible to compute standard colour coordinates from a knowledge of the distribution of energy in the stimulus concerned. The principles followed have been discussed at some length in Section II when we were describing calculations carried out on the system of Resolution (1), they are mentioned again in the discussion on Resolution (5) in the modified form for the standard system, in Section IV we have shown explicitly how the calculations are to be arranged on the standard system, and we have prepared special tables (VI, VII, VIII) which themselves illustrate the application of the method to the standard illuminants *A*, *B*, *C* respectively.

The accuracy of the coordinates obtained by this method depends on the accuracy and the sufficiency of the initial determinations of the energy distribution.

If these are inaccurate or inadequate the resulting coordinates can only be reasonably correct by chance. If we are dealing with a source of unknown energy distribution the real problem becomes the measurement of that energy. When light from a known source is transmitted or reflected by a coloured material the real problem is the spectrophotometry of the material.

If in the first place we consider real sources we have to recognise three different classes. The first and simplest consists of sources which distribute the energy according to a known law. As many sources behave in the visible spectrum as though they were grey bodies, or nearly grey bodies, Planck's law is of special importance. Carbon filament lamps may be regarded for colorimetric purposes as grey bodies, provided the bulb is free from deposited matter. When the colour temperature of such a lamp is known the energy distribution is also known, and the indirect method is easily applied. Standard illuminant *A* was computed in this way. The lamp in this case is a tungsten, not a carbon filament, and the theoretical formula does not apply so closely, but the departures are not of colorimetric importance.

Sources such as Nernst filaments, gas mantles, carbon arcs, and filament lamps enclosed in tinted bulbs form an intermediate class. They give continuous spectra in which the distribution of energy cannot be computed with adequate accuracy from a formula. The most satisfactory method, at the present time, of finding the energy distributions of such sources is to make spectrophotometric comparisons, at a sufficient number of wave-lengths throughout the spectrum, with a lamp which follows the grey-body law. Direct spectroradiometric measurements are less satisfactory. The technique of these measurements will have to be greatly developed before this method can supersede the comparative method.

The third class of source includes those which yield line spectra. The difficulties of finding the energy distributions of these sources with adequate accuracy for colour measurements are very great, even when the method of spectrophotometric comparison with a source of known energy distribution is employed. Although these difficulties may be overcome by an experimenter well versed in these measurements, the colorimetry of sources of this class by the indirect method is unlikely to become of immediate importance.

We now turn to the properties of materials. The determination of the colours of transparent materials by the indirect method, particularly those of filters, whether liquid or solid, used in colorimeters, is likely to be of immediate importance. The principles governing these measurements are well known, and it is only necessary to state here that a good standard of accuracy is essential. Crude results are quite inadequate, and it is thus necessary to use well-designed apparatus.

The spectrophotometry of reflecting materials is apt to be difficult on account of insufficiency of light. At the present time it is probable that the indirect method will only be applied to materials which have high reflection factors for the whole visible spectrum.

We may summarise by stating that the utility of indirect colorimetry is at present practically confined to sources of or nearly of the black-body type, and to light from sources of this class after passage through a transparent filter. Most of the calculations required will relate to colours seen under one of the standard illuminants A, B, C and after the experimental work is completed Tables VI, VII, VIII should be used to reach the final specification.

XII. HUE-WAVE-LENGTH AND COLORIMETRIC SATURATION

Colours are sometimes expressed by giving the hue-wave-length and the colorimetric saturation. The colour is regarded as a mixture of a standard white and a spectral colour. In the specification of some colours on this system the amount of the spectral colour is negative, that is to say the spectral colour must be added to the colour under measurement to match the white. The colour is specified by the wave-length of the spectral component—called the hue-wave-length—and by the colorimetric saturation, which is defined as the proportion of the spectral component in the mixture when the spectral and white units are of equal luminosity.

It is necessary to specify the standard white of the specification to give the figures any meaning. In what follows we assume that the equi-energy distribution is adopted as the white. The modifications in the equations when another choice is made will be obvious. To transfer from this hue and saturation system to the standard trichromatic coordinates is readily effected by using Table IV, which gives the properties of the standard observer in detail. Let λ be the given hue-wave-length, and σ the colorimetric saturation. Also let x, y, z be the standard coordinates to be found, and x_λ, y_λ, z_λ those for wave-length λ obtained, by linear interpolation if necessary, from the table. Then since the coordinates of the "white" we are concerned with are $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{3}$, the coordinates of any point which can be matched by a mixture of this white and light of wave-length λ satisfy the conditions

$$\frac{x-\frac{1}{3}}{x_\lambda-\frac{1}{3}}=\frac{y-\frac{1}{3}}{y_\lambda-\frac{1}{3}}=\frac{z-\frac{1}{3}}{z_\lambda-\frac{1}{3}}=k, \text{ say.}$$

Now the luminosity of each trichromatic unit is measured by its coordinate y, and to obtain units of equal luminosity we therefore divide the trichromatic unit by its y. The definition of the colorimetric saturation then gives

$$\frac{1}{y}=\frac{\sigma}{y_\lambda}+\frac{1-\sigma}{\frac{1}{3}}$$

or

$$\frac{y-\frac{1}{3}}{y_\lambda-\frac{1}{3}}=\frac{y\sigma}{y_\lambda}=k$$

or

$$\frac{1}{k}-1=3y_\lambda\left(\frac{1}{\sigma}-1\right).$$

This gives k, and hence x, y, z.

When the converse transformation is required an obvious method is to use a large-scale graph of the spectral locus and find the hue-wave-length and the coordinates of this spectral point by using the collinear law. The value of k and hence that of σ are then easily found.

The principles of the graphical solution of this and similar problems are described in a paper by Guild (10) who shows how the calculations can be carried out rapidly with ample accuracy for colorimetric purposes. It is easy to obtain, by the graphical method, a precision of $\pm\, 0{\cdot}1\,\mathrm{m}\mu$ in hue-wave-length and $\pm\,\frac{1}{4}$ per cent. in colorimetric saturation. Higher precision is not of visual significance. Sometimes, however, it may be desired, for the purpose of coordinating specifications on different systems, to obtain still closer values of the hue-wave-length and colorimetric saturation corresponding to a given trichromatic specification.

We can carry out the conversion entirely algebraically in various ways. It will suffice if we give one method. We have tacitly assumed above that for short spectral intervals the wave-length varies uniformly along the locus and that the locus is effectively straight between successive tabulated points. On the standard colour chart (Fig. 3) let E represent the equi-energy distribution, P the given colour, K the point in which EP meets the spectral locus, and I, J the tabulated spectral points nearest to K.

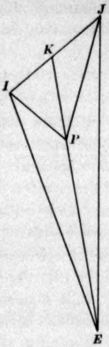

Fig. 3.

Considering triangles on the common base PE, we have

$$\frac{IK}{KJ}=\frac{\triangle PIE}{\triangle JPE},$$

and also taking triangles on the common base IJ,

$$\frac{KP}{KE}=\frac{\triangle JIP}{\triangle JIE}.$$

The former ratio gives λ by interpolation between the values at I and J, and the latter is $1-k$, and leads to σ by

$$1-\sigma=(1-k)/3y.$$

The area of any triangle is proportional to the determinant formed from its coordinates. In practice it is most convenient to adopt a special form involving only two of the three variables. Suppose that of the three coordinates x, y, z of the given colour, z is the one which differs least from $\frac{1}{3}$. Let

$$\phi_{PQ}=(x_P-\tfrac{1}{3})(y_Q-\tfrac{1}{3})-(x_Q-\tfrac{1}{3})(y_P-\tfrac{1}{3}).$$

Then the area of the triangle PQE is a constant multiple of ϕ_{PQ}. Thus if P represents the known colour (so that the suffix P can be dropped),

$$\begin{aligned}\triangle PIE &= c\phi_{PI}\\ \triangle JPE = -\triangle PJE &= -c\phi_{PJ}\\ \triangle JIE &= -c\phi_{IJ}\\ \triangle JIP &= -c\,(\phi_{IJ}+\phi_{PI}-\phi_{PJ}),\end{aligned}$$

or

$$\frac{\lambda-\lambda_I}{\lambda-\lambda_J}=\frac{\phi_{PI}}{\phi_{PJ}},$$

I and J representing the tabulated wave-lengths between which ϕ changes sign, and

$$k = \frac{\phi_{PJ} - \phi_{PI}}{\phi_{IJ}}.$$

The equation giving σ in terms of k has already been found.

The chief weakness of this method is that I and J cannot be selected from the inspection of Table IV. A graph, even if comparatively rough, is of great convenience to avoid useless calculation. It should also be observed that, since the ϕ's are small quantities, figures should not be dropped in the products unless it is known that this can be done with impunity.

APPENDIX

When considering the advantages which we should like to secure in a standard colorimetric scheme we stated that some of these were not obtainable in an otherwise satisfactory system. We give here an analysis which may help to make the ground for these conclusions apparent to others.

Let X, Y, Z be the units of a trichromatic system in which all real stimuli have positive coefficients. Geometrically this means that the region of all real colours lies within the reference triangle. Since this region is bounded by the spectral locus (which is everywhere convex) and the straight line joining its extremities, it is obvious that the only real colours we can take for the corners of the triangle are the extremities of the visible spectrum, and that the third stimulus is necessarily unreal.

If, in order to have two real unitary stimuli, we take these spectral extremities as two of the corners, X and Y, of the reference triangle, the region within which the third unitary stimulus may lie is determined by the tangents to the spectral locus at its two extremities, for if any side of the triangle cut this locus in another point, part of the locus would lie on one side of this line and part on the other, that is to say some coordinates would be positive and others negative, thus contravening the original stipulation. From Fig. 4, which shows the spectral locus on the system of Resolution (1), it is clear that the third angular point Z on this chart will lie on the side of XY away from the real colours. In the equation

$$Q = xX + yY + zZ$$

of any real colour the coefficient z will therefore be negative. This may be disguised by writing $-z$ for z and $-Z$ for Z; in effect we have now turned the triangle inside out, and brought within the triangle the space bounded by XY, ZX produced, and ZY produced, formerly outside the triangle. The region of real colours thus comes within the triangle. But in representing this infinite stretch of the original plane on the finite triangular area we have subjected parts to very great compression. The scheme fails because the compression suffered by the parts of the spectral locus more remote from XY involves such great distortion that the chart becomes misleading rather than helpful. We either have the spectral locus confined to a small part of the new triangle—and this is obviously objectionable—or, by making it occupy a large part, we force the locus to assume approximately the shape of the triangle itself, thus giving the false impression that all real colours can be effectively matched by the mixture of three particular spectral colours. We therefore have to abandon the idea of using two real unitary stimuli.

A homogeneous transformation such as that represented by equations (12) involves no distortion. It gives in place of the numbers u, v, w other numbers x', y', z'. The determinant formed from three sets of u, v, w is a fixed multiple of the corresponding x', y', z' determinant. The distortion is introduced by the unifying factors, and in a small triangle the area is affected by its cube. It is easy to see that in one direction the first power, and in the perpendicular direction the square of the unifying factors, is operative. This is a well-known result for collinear transformations, and is illustrated by the laws of optical imagery. If therefore we start from a system which is reasonably satisfactory, we shall not derive a satisfactory system from it if the ratio of the extreme unifying factors differs greatly from unity. For instance starting from the system of Resolution (1) this factor should not exceed 2, or 3 at the outside. To form an idea of the magnitude of the factor in a simple case let us neglect the blue extremity of the spectral locus on Resolution (1) and take as the vertex of the new triangle the point Z in Fig. 5. From the geometry of the figure, remembering that the coordinates represent the areas of triangles, we obtain $x' + v = y' + u = 1$, $w + z_1' = 0$. To disguise the fact that z_1' is negative, we change the signs of both Z and z_1'; thus putting $z' = -z_1'$, we obtain

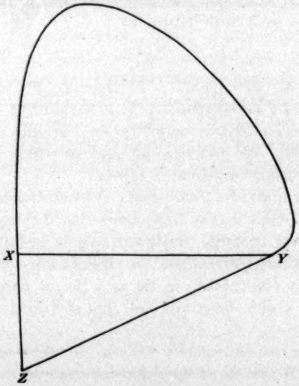

Fig. 4.

$$x' = 1 - v, \quad y' = 1 - u, \quad z' = w$$

and applying the conditions $u + v + w = x + y + z = 1$, we obtain

$$x = \frac{1 - v}{1 + 2w}, \quad y = \frac{1 - u}{1 + 2w}, \quad z = \frac{w}{1 + 2w}.$$

Now from Table I we see that w varies roughly from 0 to 2, and the unifying factor $1 + 2w$ therefore changes in the ratio 5 : 1. In some direction therefore the distortion is represented by a value of about 25 : 1, and this is far greater than any admissible amount.

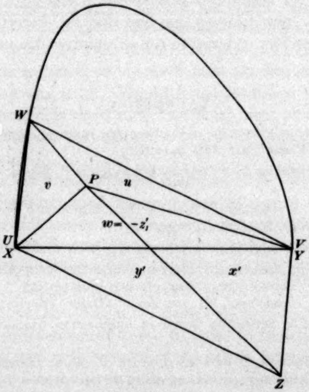

Fig.5.

In this illustration we have not taken all the important factors of a transformation into account. By using the general transformation equations we readily find that to transform

$$\begin{aligned} &(1\ 0\ 0) \text{ to } (1\ 0\ 0)\\ &(0\ 1\ 0) \text{ to } (0\ 1\ 0)\\ &(u_1\, v_1\, w_1) \text{ to } (0\ 0\ 1)\\ &(u_0\, v_0\, w_0) \text{ to } (\tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}) \end{aligned}$$

we obtain the transformation equations

$$x' = \frac{w\, u_1 - u\, w_1}{w_0 u_1 - u_0 w_1}, \quad y' = \frac{w\, v_1 - v\, w_1}{w_0 v_1 - v_0 w_1}, \quad z' = \frac{w}{w_0}.$$

The numerators of x' and y' merely indicate that the sides of the new triangle when projected on the original chart must not intersect the spectral locus. The unifying factor is essentially

$$k = 3w + w_1 \left\{ \frac{w\, u_0 - u\, w_0}{w_0 u_1 - u_0 w_1} + \frac{w\, v_0 - v\, w_0}{w_0 v_1 - v_0 w_1} \right\},$$

and to show its dependence on the selection of u_1, v_1, w_1 or u_0, v_0, w_0 we may take $u - v$ and w as independent variables. Thus for example we find

$$\frac{dk}{dw_1} = w_0 \left\{ \frac{(wu_0 - uw_0)(1 + u_1 - v_1)}{(w_0 u_1 - u_0 w_1)^2} + \frac{(wv_0 - vw_0)(1 - u_1 + v_1)}{(w_0 v_1 - v_0 w_1)^2} \right\},$$

showing that w_1 (which in our problem is necessarily negative) should be as large numerically as possible, i.e. it is best from the point of view of reducing distortion to take the two tangents to the spectral locus as two sides of the new triangle as shown in Fig. 4. If we adopt this principle and take the equi-energy spectrum as the centroid of the new triangle, we find that the distortion is too great to be admitted. We are therefore reduced to the alternatives discussed on Resolution (5) in Section II.

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DISCUSSION

Dr W. D. Wright: This paper is a most valuable and important one, and it is to be hoped that its significance will be fully appreciated. It is significant because it describes a landmark in the history of colorimetry, and it is valuable as a detailed account of the processes by which the standardised data were obtained.

It is, perhaps, somewhat unfortunate that, in the first publication of these data, they should be subjected to such a detailed analysis of their properties and inter-relations. It should, I think, be the aim to simplify the subject wherever possible and to encourage the introduction of trichromatic colorimetry into industrial laboratories and, as these standard data are an essential part of colorimetric equipment, it is a pity that they have been linked up with a good deal of algebra that is only occasionally of interest to the technician. I do not suggest that the algebra is unimportant, but that it would have been better discussed in a separate paper. It would have been more appropriate, to my mind, to have followed up the description of the resolutions by an outline of the relatively simple computation necessary to obtain the specification of a colour in terms of the standardised units from measurements made on a trichromatic colorimeter.

There are only one or two points of detail on which I would like to comment. One relates to the suggestion that the various industrial laboratories concerned should investigate the value of all three illuminants, in the hope that one of them will become of more predominant use than the others. This is a much desired goal, but it is to be hoped that the necessity to try three different illuminants will not act as a deterrent against using any at all. Would it not be better to suggest that each laboratory used the source that most suited its needs, and, as the number of users increased, to find whether the advocates of one source greatly outnumbered those advocating the other two?

In a discussion with Mr Priest after the meeting of the Committee, a point was raised regarding the illumination of coloured materials, such as ribbed cloths, in which the surface structure varies in different directions. Is there any standard practice for orienting the cloth with the rib vertical or horizontal or at some given inclination? It is a small point but one on which, I presume, a ruling will have to be given in due course and might, perhaps, have been included in Resolution (3).

I think it should be mentioned that the device adopted in Tables VI, VII, and VIII, in which the $E\bar{y}$ ordinates sum to 100 in order to facilitate the calculation of percentage brightnesses, can only be used if the values for any given material are tabulated for all the wave-lengths given in these tables. For fairly uniform reflection the wave-length intervals may be found unnecessarily small.

As regards the use of the terms unitary, basic, and cardinal stimuli, the only objection is that it may add to the confusion of the subject to inject more terms into it, particularly if it should prove that the term primary is too old-established to disappear readily. I think the amount of misunderstanding that clings to this term is probably overestimated, but I am personally prepared to give the new terms a fair trial, with as unbiased a mind as possible.

In conclusion, I would like to express my personal admiration for the enormous amount of work done by Mr Guild and Mr Smith in a very short time, when faced last September with the prospect of a postponement of international agreement for a further three years.

Prof. A. O. Rankine: I am interested to know whether there exists any means regularly available for testing how nearly an observer is "standard." A new observer may be required by a firm for colorimetric work. How is it to be ascertained that his observations will be reliable?

Mr J. W. Perry: The appearance of this paper marks an epoch in the history of colorimetry and the authors are to be congratulated on the completion of the lengthy computations involved in the work here recorded. Colorimetry has hitherto lacked universally accepted accurate data and progress will be considerably facilitated by even provisional acceptance of the system of colorimetric normalisation here recommended. In view of this momentous and fundamental character of the paper, I feel it is not a case where rein should be given to criticism except in accordance with actual practical experience with the proposed system over a reasonable length of time. It is regrettable that no compromise could be effected in the decisions relating to the standard illuminants *B* and *C*, enabling these to be supplanted by a single illuminant.

I hope this may also be regarded as determining an unique colorimetric notation. On the subject of the interpretation of Resolution (3), referred to by Dr Wright and applying to the case of the colorimetry of special substances having peculiar directional properties of surface reflection or transmission, I think this is a case of classifying the test object and employing a special technique for special categories of materials, rather than attempting in a form of words to anticipate all such cases that may possibly arise in practice.

Mr Smith: I will reply to one or two of the general points mentioned in the discussion and leave others to those who can speak with wide experience of the experimental side of the subject.

Some of the comments assume that the fundamental purpose of the paper is to smooth the path of the technical worker, with practically no previous knowledge of colorimetry, who wishes to be able to make use of the standards after the least possible study of the subject. While we should all admit the importance of directions of the utmost simplicity for the guidance of industrial workers, I do not agree that the first paper on the new standard should be of this character. When new standards are set up, the bodies primarily concerned are standardising institutions, and the first requirement is to record the experience already gained for the benefit of those who, in various countries, may be called upon to calibrate instruments on the new system. The correctness of this view is borne out by the fact that the only requests for fuller information we have so far received have come from well-known laboratories, and have related to the parts of the subject which Dr Wright thinks it would have been better to omit.

The terms unitary stimuli and cardinal stimuli are to be regarded at present as suggestions. We have found them convenient in writing this paper, and our experience may agree with that of other workers. Had we not felt the objections to "primary" to be decidedly weighty, we should not have sought for a new word. The need for a special word bearing the significance we have given to "cardinal" only effectively arose with the adoption of Resolutions (4) and (5).

The standardisation of more than one illuminant has been adversely criticised, and it has also been suggested that advice should now be given to use one illuminant in general, leaving the others for abnormal work. In the first place, I would say that I do not share the fears that the mention of more than one illuminant is a complication which will tend to delay standard colorimetry. Doubtless many new users will for a time confine themselves to one illuminant, but when the routine is thoroughly understood the other illuminants will in many cases be tried. At the present time there is no doubt at all that the ultimate adoption of one illuminant as a predominant international standard depends entirely upon experience with all the present standards in industrial colorimetry. The immediate tendency is for one illuminant to be favoured here and another in America. A permanent cleavage of this kind is not likely to be helpful to the industries most concerned with standard colorimetry. The Bureau of Standards and the National Physical Laboratory have agreed for the present to treat all the illuminants on an equal footing. This leaves the industries of both these and of other countries free to determine, on the basis of their own experience, whether both sources are needed, or only one is generally necessary, or possibly that a single intermediate source would be preferable. When the subject comes up for reconsideration in 1934, it is not to be expected that opinions which have no basis in comparative experimental work will carry much weight. I would therefore emphasise the importance of the advice given in the paper to gain experience with all three illuminants.

The best orientation of rough textiles is a problem special to a particular industry, rather than a general colorimetric question. It would most appropriately form the subject of agreement within this industry itself. It should be remembered that Resolution (3), like the others, does not lay down a procedure necessarily to be followed in every case. It describes the normal procedure; when any other procedure is followed it must be specially described.

Mr R. Donaldson: In reply to Prof. Rankine's enquiry, the question of the relationship of any observer to the normal observer does not arise until the normal observer has been exactly defined. As this has only just taken place at the Congress, no regular process of testing the normality of an observer has yet been devised and agreed upon.

A considerable amount of information on this point may be obtained, however, by direct measurements on a small number of test colours. These colours should be fairly saturated and occupy positions on the colour chart approximately midway between the primaries of the measuring instrument. The values of the colour coefficients which apply to the normal observer can be obtained by calculation from the spectrophotometric curves and the energy distribution of the illuminant. A comparison of the measured and calculated values will give an indication of the extent of the observer's departure from normality.

Mr Guild (communicated): From the point of view of the psychological reaction to this paper of technical workers in the colour industries the point raised by Dr Wright is important. We gave considerable thought to it in planning the paper, and considered that the most generally useful course was to combine in one publication an account of the derivation of the new standards with all information required for their practical utilisation. We endeavoured, however, to segregate as far as possible those matters which are of importance to the technical worker who is merely concerned with obtaining colorimetric results on the standard system either by the use of a trichromatic colorimeter or by computation from spectrophotometric data.

Any such worker whose interest is confined to the immediate requirements of practical colorimetry need not trouble to become familiar with more than a few sections of the paper. In particular, he need not concern himself with any mathematical formulae; these are only required by those who may be carrying out the actual standardisation of colorimeters. The ordinary technical worker will normally use an instrument already standardised at the National Physical Laboratory or at an equivalent institution abroad. The results of such calibrations can be given to him in a form requiring, on his part, only simple arithmetic in order to convert his observed results to the standard form. British colorimetrists have been familiar with this process for several years past, and will find that the technique which they have followed is not appreciably affected by the new standards. They are not called upon to perform any additional process in their experimental work nor in the routine calculations connected with it. Temporary unfamiliarity with the units in which their final results appear and with the form of the spectral locus in the new colour triangle will soon wear off.

With regard to another point raised by Dr Wright, that for some specimens the wave-length intervals of 5 mμ for which the data are tabulated are unnecessarily small, it should be pointed out that Tables VI, VII, and VIII were prepared, in the first instance, for our own use at the National Physical Laboratory, and are included in the present paper because they may be of use to others engaged on similar work. My personal view, with which, I think, Dr Wright is familiar, is that colour computation from energy distribution measurements is, at the best, too tedious to be of much importance as a routine method of determining the colours of materials. Its special province, in my opinion, is in fundamental standardisation work where, for various purposes, it may be desired to obtain standard values for suitably selected light sources or material specimens, either for checking the direct colorimetric results of observers, or in standardising colorimeters of various types. In such work it is always desired to obtain the greatest possible precision in the numerical results, and the use of the full tables is necessary.

Mr Donaldson has explained, in reply to Prof. Rankine, the procedure we would at present adopt for checking the suitability of an observer for practical colorimetry. I may add that we are at present investigating the problem of reducing the personal errors of observers by means of correcting filters. Mr Donaldson has already done considerable preliminary work on this problem, and we hope to bring it to a satisfactory issue in the near future.

DEMONSTRATION OF THE MEASUREMENT OF PARTICLE SIZE AND CONCENTRATION IN OPAL GLASSES

By C. DUNBAR, M.Sc.

(Photometry Division, The National Physical Laboratory)

Demonstrated, 10th March, 1932.

A SAMPLE of opal glass, which contained diffusing particles of diameter 1 micron, was examined with a microscope employing an oil immersion objective, the magnification in the final image plane being approximately 5000 diameters.

The relative size of the particles in the image plane can be measured on some arbitrary scale which has subsequently to be reduced in terms of the magnification of the optical system.

The value of the magnification is obtained, not by the use of a test object of known size, but by a measurement of a displacement of the mechanical stage, since the image of any object on the stage is displaced in the image plane by an amount equal to the product of the stage displacement and the magnification of the system.

The microscope is fitted with a specially designed mechanical stage, which can be moved through very small distances by means of a differential screw. The fixed and mechanical stages carry plates of a Fabry-Perot interferometer system, the distance between the plates changing as the mechanical stage is moved. Since the passage of one fringe over the cross-wire of the viewing telescope corresponds to a half wave-length change in the distance between the plates, the displacement of the mechanical stage can be obtained directly in terms of the wave-length of the light illuminating the plates.

The concentration of the particles is found by focussing through a thickness of the opal glass which is large compared with the focal depth of the microscope objective, and counting the number of particles which appear in a limited field. The depth through which the microscope is focussed is measured by a second interferometer system with one plate on the barrel of the microscope. A focussing rate of about a wave-length a second can be obtained by driving the vernier wheel of the microscope with a slow clockwork motor. Since the diameter of the limiting aperture can be found in the same way as the particle size, the number of particles in a known volume is determined.

The concentration of the particles in the opal glass viewed was approximately $5 \times 10^{10}/\text{cm.}^3$.

REVIEW

Modern Cosmologies. By HECTOR MACPHERSON, M.A., Ph.D. Pp. 140; 12 Figs. (Oxford: University Press; London: Humphrey Milford, 1929.) Price 7*s*. 6*d*. net.

Cosmological speculations are as old as, and probably older than, history itself, and the story of their development is doubly fascinating—alike by reason of their intrinsic interest, and of the manner in which they illustrate the sweep of, and indeed in some of their aspects certain aberrations of, the human intellect. It is a far cry from the homely anthropocentric cosmogony of Anaxagoras, with a sun about the size of the Peloponnesus to the universe recently limned by Eddington—a universe of blank regions where spiral nebulae populate space as thickly as two or three wasps might crowd York Minster which yet, in desperate haste to relieve this pressure of over-population, is expanding at such a rate that we shall be compelled to double the apertures of our telescopes every 1300 million years in order to keep up with the recession of the nebulae.

Dr Macpherson develops his thesis in a practical and interesting manner. Early cosmogonies are briefly handled and it is not until we arrive at the later years of the 18th century that the story is developed in any detail. Sir William Herschel's great work is described very fully and critically—and here the author is to be congratulated on his attack on a problem of no small difficulty. Herschel's views concerning the structure of our universe are by no means easy to comprehend in their finer details. Naturally they changed and developed during his long and active life and as they were made public in isolated papers published through the medium of the *Philosophical Transactions* it is not surprising that his views were difficult to survey as a whole and were subject to some measure of misrepresentation. When we attempt, however imperfectly, to survey the vast mass of literature, good, bad, and indifferent, that has poured from the presses of the last century and a half, our first emotion is apt to be one of thankfulness when we hear of an author who is content to leave his work buried in the decent obscurity of a learned publication. But it is a matter for real regret that Herschel never published his views in book form—it would certainly have saved some misunderstanding, and might possibly have accelerated the development of later views.

These later views are discussed very simply and straightforwardly and the manner in which estimates of the scale of our universe have waxed and waned and waxed again is clearly set out. The treatment is descriptive, and even though the subject is very well developed without much reference to atomic and electronic doctrine, a discussion of these matters would add much to the usefulness of the volume.

Just now cosmological doctrine is in a state of rapid flux, and Dr Macpherson is hardly to be criticised for giving us none of the consequences of Lemaitre's paper of 1927. The paper was not easy of access; now it is fortunately available in the *Monthly Notices*. In a new edition of the author's very useful work, which will, we trust, be called for at an early date, we hope to find an account of the expanding universe as graphic as the accounts which the author has given of those cosmologies which, even in the telling, are as a tale that is told.

A. F.